

Part One

Truth Paradoxes and Varieties of Meaning

Abstract

The aim of this essay is to show 1) that there are various types of truth paradox exhibiting various kinds of *prima facie* unacceptable consequences derivable in a *prima facie* appropriate manner from *prima facie* indisputable assumptions, 2) that in all those cases the derivations rely on some shared principle(s) linking meaning to truth without which the seemingly indefensible results would not follow, 3) that considerations whose plausibility is not exclusively based on the fact that their acknowledgment would eliminate a basis for the derivation of paradoxical consequences support a dismissal, or rather a modification, of the crucial principle(s), 4) that, viewed in this light, truth paradoxes do not provide compelling reasons for rejecting established derivation rules of (classical) logic or for discarding common intuitions about truth and 5) that certain linguistic items can consistently be classified as untrue or even false which otherwise are apt to be regarded as falling into a truth-value gap.

It emerges as a result that, rather than putting into question ordinary intuitions about *truth*, truth paradoxes affect our conceptions about what linguistic items *mean*, what they express and what can be asserted by employing them and indirectly even concern the concept of translation. A major outcome of our investigation lies in the circumstance that it highlights the significance of raising questions about meaning which do not (directly) pertain to linguistic expressions. They are *not* represented by questions of the form ‘What

does E mean?', where 'E' is a placeholder for a term designating a (meaningful) linguistic item of any category and where 'mean' covers all varieties of meaning which linguistic expressions can exhibit. They are conveyed rather by questions of the forms 'What does being F mean?', 'What does it mean (for x) to be F?', or 'What does it mean that x is F?'

The present essay is not concerned with the issue whether particular *formal systems* permit one to derive as theorems statements engendering truth paradoxes or how the derivability of such theorems can be prevented.⁴ It rather focuses on the question which consequences can or should be drawn from the fact that many *natural languages* permit the formulation of sentences which in virtue of their content are apt to generate truth paradoxes.

⁴ This question is dealt with in several publications on truth paradoxes: see, *inter alia*, Cook 2009:273ff.

I. Introductory Remarks

In the context of the present essay the term ‘*paradox*’ is taken to apply *primarily* to *propositions* that exhibit the following characteristics: i) It appears *prima facie* impossible to consider them as true; ii) they result as consequences from *prima facie* undeniable assumptions by applying only principles of derivation whose validity seems indubitable. In a *secondary* sense the word is also used to characterize an *argument* the conclusion of which is a paradox in the primary sense or to a totality of assumptions and principles entailing a paradox in the primary sense or to an apparently impeccable derivation where the mere fact that the conclusion, although its truth is conceivable, should follow from the assumptions is utterly perplexing. The adjective ‘paradoxical’ can be applied not only to possible contents of linguistic items, in particular sentences, but also to the linguistic items themselves that represent the crucial consequences and principles of reasoning. In addition to this, the word can even be applied to the *fact* that *prima facie* unbelievable consequences can be deduced from apparently acceptable assumptions by apparently acceptable principles of reasoning.

The elucidation just given tallies with a quite common use of the expression ‘paradox’. It is reflected, among others, in the explanation presented in Sainsbury 1988:1, according to which a paradox is ‘an apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises’. There are alternative proposals that differ from the one outlined above in various respects, e.g. by assigning the opposite priority ordering to the different employments of the term ‘paradox’. But since these divergences are immaterial for the

issues to be considered in this essay, we can confine ourselves to merely pointing out this fact without going into further details.⁵

One can make some general distinctions based on the character of the consequences which are classifiable as paradoxes. Thus we can classify a paradox (in the primary sense) as an ‘antinomy’ if and only if it consists in a proposition which is a formal contradiction (a sentence of the form ‘p and not p’).⁶ We can use the expression ‘logical paradox’ with respect to situations where the truth of a derivable outcome can be regarded as impossible due to logical reasons even though it is not a formal contradiction, for example consequences of the form ‘a is F if and only if a is not F’. Moreover (following Quine) a distinction can be drawn between ‘veridical’ paradoxes, cases in which the resulting consequence(s), notwithstanding (possible) appearance to the contrary, turn(s) out to be true as a result of deeper reflection, and ‘non-veridical’ or ‘falsidical’ paradoxes, where the consequence(s) cannot be established as true in the final analysis and might be even demonstrably false.⁷ The subsequent investigations will suggest that the differences we just mentioned are by no means the only ones which are pertinent in this area.

Before presenting an explanation of the less comprehensive term ‘*truth paradox*’ it is appropriate to introduce a notion which will be designated by the expression ‘truth comment’, or, for short, ‘T-comment’. This expression will in the first place refer to any statement or vehicle of a statement which is or implies an ascription of truth or lack of truth to any item which could be regarded as a possible bearer of the property of being true. In an extended manner the term should encompass also cases where propositions are implied relating to possession or lack of truth as their component, for example statements to

⁵ Künne 2013:40, footnote 66, points out that some authors consider as primary the usage which is treated here as extended or secondary. Regarding the possibility of a less restrictive employment of ‘paradox’, see M. Clark 2012:xi [2012:11] and 2012:159ff [2012:179ff]. — The numbers in squared brackets refer to the pages in the German edition.

⁶ Cf. Künne 2013:40.

⁷ Cf. Quine 1966:3ff.

the effect that something is true if and only if p , and similar ones. Whenever an apparently unacceptable or perplexing consequence can be derived in an apparently acceptable manner from a truth comment or a group of items to which a truth comment belongs such that the same consequence would no longer follow if among the assumptions at least one element which is a truth comment were dropped, we consider the consequence as a truth paradox, and in an extended sense we might call an entire argument exhibiting the corresponding quality — or even the fact that such an argument is possible — a ‘truth paradox’. It is implicitly presupposed that the pertinent consequences in situations of truth paradox do not depend on any assumptions which are taken as untruths in the given context and that the premises constituting the basis of the derivation characterize situations whose possible existence can be plausibly assumed. In the present connection a most important fact is that the possibility for a proposition or an argument to qualify as a truth paradox does not require that the consequence relates in any manner to matters of truth, in particular that in a sentence expressing the consequence contains the expression ‘true’ or some synonymous term. Apart from the quality of appearing unacceptable with respect to the consequence it matters only that in the set of assumptions elements occur which exhibit the character of T-comments according to the preceding explanation of the term.

The concept of a truth paradox ought by itself not embody any prejudgment concerning the issue as to what are possible bearers of truth or which sorts of items are to be regarded as primary bearers of truth in the final analysis, and the preceding explanations should comply with this desideratum. At any rate, the elucidation presented above leaves room for many variations as far as the character of pertinent assumptions is concerned, in particular the character of premises which could be considered as T-comments. For example, assumptions of the form ‘(sentence) s is true’, ‘ s is untrue’ as well as assumptions of the form ‘NN said something true’ or ‘NN is right in asserting that p ’ etc. could (in principle) function as premises for the derivation of a truth paradox. The range of linguistic expressions which can be considered as T-comments is pretty diverse embracing,

among others, expressions like a) 'yes', 'no', if the words are employed for claiming or disclaiming truth, b) expressions of the form 's expresses a truth', 'what s expresses is not true', where 's' relates to some linguistic item, c) 'A does not say anything which is true', 'what A said is true', where 'A' refers to a being capable of speaking a language. It deserves to be emphasized, on the other hand, that sentences or statements attributing the act of lying or the property of being a liar to an agent, or the property of being a lie to an utterance should not be regarded as members of the pertinent realm. The reason is that it is highly doubtful that lies have to be untruths.⁸ Accordingly, we ought to reckon with the possibility that strictly speaking there are no liar paradoxes.

Although the area of truth comments and accordingly the potential realm of truth paradoxes is more encompassing, the subsequent exposition will draw the attention in the first place to situations where pertinent assumptions exhibit the forms of a) 'what s Φ s is (not) true / is (not) a truth' and b) 's Φ s something which is (not) true / is (not) a truth' or closely related ones, where 's' refers to a linguistic item, such as a sentence (token), and ' Φ s' should be replaced by a verb signifying a relation between a linguistic unit and some (sort of) content associated with that unit, as for example 'expresses', 'conveys' etc. The reason lies in a supposition to the effect that the central results obtainable from considering these varieties of T-comment can be transferred to the whole realm.

Even under this provisional restriction a significant number of varieties have to be acknowledged due to the variation of the nature of the consequences qualified as truth paradoxes in what we have called the primary sense of this term. For this purpose, it is apposite to draw attention first to a pair of principles which appear at first glance pretty plausible. They have been recently presented in a version which relates to speakers of a language and read as follows:

⁸ See Künne 2013:24ff (chapter 1, §2) and Oetke 2013:11–12.

[P1] if (x says that p and x thereby says something which is true) then p.

[P2] if ((x says that p) and p) then x says something which is true.⁹

The intuition underlying these principles can be equally represented by two related principles which involve reference to linguistic items and might be formulated thus:

(TE1) [For any linguistic unit s:] if (s expresses that p and s thereby expresses a truth) then p.

(TE2) [For any linguistic unit s:] if (s expresses that p, and p) then s thereby expresses a truth.

These principles appear *prima facie* incontestable at least as far as sentences which are free from ambiguity are concerned.¹⁰ If one acknowledges these principles it is most natural to accept two derivation rules as valid. The first rule licenses the deductive step from two assumptions to the effect that a linguistic unit expresses a truth and that it expresses that p (and nothing else) to the consequence that p. The second one legitimizes the step to the conclusion that some linguistic item expresses a truth from the assumptions that p and that the linguistic item in question expresses that p. In both cases the consequence would depend on no other assumptions than those on which the two assumptions depend (given that assumptions introduced

⁹ Künne 2013:40. In the original German version they read:

[P1] wenn (x sagt, dass p, und x sagt damit etwas Wahres), dann p.

[P2] wenn (x sagt, dass p, und p), dann sagt x etwas Wahres.

The brackets in the English [P2] like the comma preceding ‘und’ in the German original are to prevent the unintended reading ‘x says that (p and p)’.

¹⁰ The occurrence of ‘thereby’ is in fact dispensable with respect to items free of ambiguities. In Oetke 2012:253 the two principles had been obtained by the decomposition of a theorem which reads:

[For any linguistic unit s:] if s expresses that p, then s (thereby) expresses a truth if and only if p.

— The expression ‘E*’ occurring there (on p. 246 and later) has been replaced by ‘E’ in the present investigation and can be read as ‘expresses something which is true’.

in a proof depend on themselves). Accordingly, one might specify the following rules of derivation which, even if they are not commonly mentioned as ‘logical derivation rules’, could perform analogous functions (the turnstile $\vdash$ can be read as ‘therefore’):

(EE) s expresses a truth; s expresses that $p \vdash p$

(EI) p ; s expresses that $p \vdash s$ expresses a truth

The designations ‘EE’ (*Expressing Elimination*) and ‘EI’ (*Expressing Introduction*) are meant to reflect the circumstance that in the first case any reference to expressing a truth is ‘eliminated’ in the conclusion, whereas in the second case such reference is ‘introduced’.¹¹

¹¹ As one of the referees noted, these rules are valid if, and only if, substitution instances of ‘ s ’ express exactly one proposition.

II. Types of Paradox

A) Simultaneous exhibition of or equivalence between truth and untruth

It seems feasible to deduce from premises which appear to represent possible facts a conclusion whose truth appears impossible to accept because it amounts to the supposition that one and the same (linguistic) item is both true and untrue, or that it both expresses a truth and fails to express a truth. In order to make as plain as possible the fact that the occurrence of this kind of situation is not affected by the question of whether one considers linguistic units as possible truth-value bearers or rather (non-linguistic) items which are in some manner related to linguistic units, we shall use the symbol 'E' in such a way that it may be interpreted either as 'is true' or as 'expresses a truth' (or in any sense which would be specified by replacing 'express' by any other term signifying a relation between linguistic entities and some sort of their content).

A rather straightforward — and familiar — way to derive an 'antinomy of truth' would be the following. Let 'Λ' designate the expression '¬ E Λ', where '¬' should be read as 'not' or 'it is not the case that' and where '¬ E Λ' means that the item designated by 'Λ' is not true or does not express a truth. This would seem to license the following steps of derivation¹²:

I)

1	(1)	E Λ	A(ssumption)
1	(2)	¬ E Λ	1 EE
1	(3)	E Λ & ¬ E Λ	1,2 &I
	(4)	¬ E Λ	1,3 RAA
	(5)	E Λ	4 EI

¹² For information about the style in which derivations are presented in this essay see the Appendix.

$$(6) \quad E \wedge \& \neg E \wedge \quad 4,5 \& I^{13}$$

Alternatively, instead of (6) one could equally derive as a consequence:¹⁴

$$(6a) \quad E \wedge \text{ if and only if } \neg E \wedge$$

Analogous consequences could be drawn without introducing stipulations concerning designations of linguistic units. A pertinent example, which reads

‘Yields a falsehood when appended to its own quotation’ yields a falsehood when appended to its own quotation.

is presented in Quine 1966:7.

This way of reasoning could be licensed by a maxim to the effect that from any (declarative) sentence an expression can be derived that results from replacing it by its own quotation or designation and attaching ‘E’ to it, and vice versa. Hence no (explicit) reference would be made to the idea of something which is expressed by linguistic items or which is related to them in some way as their content. On the other hand, if one directs one’s attention to the question of the justification of that maxim, the idea of something which expressions possess as their content emerges in a most natural way, and (TE1) and (TE2) appear to provide the corresponding intuitive basis. This fact is vindicated by the following remarks occurring in Sainsbury 1988:119:

Consider the sentence

L_2 . L_2 is not true.

This is even more immediately paradoxical than L_1 .¹⁵ Suppose L_2 is true. Then it is as it says it is, viz., not true; so it is not true. However, suppose that it is not true. Well, *not true* is just what it says it is, and a sentence that tells it the way it is is true; so it is

¹³ ‘RAA’ and ‘&I’ refer to the rules of ‘*reductio ad absurdum*’ and ‘&-introduction’ respectively.

¹⁴ Cf. Bocheński/Menne 1973:93; Sainsbury 1988:114ff.

¹⁵ Sainsbury had introduced ‘ L_1 ’ on p. 114 as a designation of ‘ L_1 is false’.

true. To sum up: If L_1 (*recte* L_2) is true, it is not true; if it is not true, it is true.

B) Conflicts with apparently incontrovertible assumptions concerning semantic content

Let us amplify the preceding sequence of derivational steps portrayed in I) by adding as an additional assumption a proposition to the effect that some sentence expresses with respect to itself that it is not true or that it fails to express anything which is true. This idea can be formulated as follows:

s expresses that not: E s

By permitting that the second occurrence of ‘s’ might be used referentially so that the particular way in which some sentence refers to itself is not essential one can consider this formulation as representing the ideas that some linguistic unit says or expresses about itself that it is not true or that it fails to express any truth — in accordance with the specific way one wants ‘E’ to be understood. There are various ways in which it could be achieved that a linguistic item attributes lack of truth or failure of expressing any truth to itself. It is not necessary to employ within a sentence any designation which one has arbitrarily assigned to that very sentence. One possibility lies in referring to linguistic tokens by using expressions of the form ‘the sentence which is uttered by NN at place P at time T (.....)’, such that the contextual features are specified to the extent that the individual token that is being produced is uniquely designated, and attaching to this singular term a predicate by which lack of truth or failure of expressing a truth is attributed. Another possibility is indicated by the Quinean strategy mentioned above. There is no need to explore various alternatives in more detail as long as it can be taken for granted that there is at least one way in which a linguistic item can self-ascribe lack of truth or failure of expressing a truth. The following derivation of a paradox requires merely this possibility.

II)

1	(1)	Σ expresses that $\neg E \Sigma$	A
2	(2)	$E \Sigma$	A
1,2	(3)	$\neg E \Sigma$	1,2 EE
1,2	(4)	$E \Sigma \ \& \ \neg E \Sigma$	2,3 &I
1	(5)	$\neg E \Sigma$	2,4 RAA
1	(6)	$E \Sigma$	1,5 EI
1	(7)	$E \Sigma \ \& \ \neg E \Sigma$	5,6 &I
	(8)	Σ does not express that $\neg E \Sigma$	1,7 RAA

Would it be justified to dismiss the relevance of this derivation because it is obtained from I) by merely adding an additional assumption and by incorporating the original steps of derivation in the new framework? To be sure, the additional assumption introduced in II) appears to make only explicit a possibility which is tacitly presupposed in I). But in contrast to I), the consequence that represents the paradox in II) is neither an antinomy nor some proposition one would be inclined to classify as a ‘logical impossibility’ in accordance with a natural understanding of that expression. It does not primarily pertain to the notion of truth, but to the issue of what linguistic units might be able to express, say or convey etc. The character of paradox results because the assumption of the first line seems to describe a possible situation and the derivation in its entirety seems to show that this possibility does not obtain. While the considerations in section A), specifically the reasoning displayed in I), are apt to arouse questions about our conception of truth, II) makes manifest that essentially the same argument could equally be understood as signalling a problem concerning meaning and in particular the relationship between meaning and truth.

Similar observations can be made against the background of items disclaiming truth with respect to themselves in ‘indirect’ ways, such as X’s attributing truth to some unit Y which in its turn disclaims truth with respect to X.

II*)

Let us suppose:

Ψ : E Ω

Ω : $\neg$ E Ψ

Thus:

1	(1)	Ψ expresses that E Ω	A
2	(2)	Ω expresses that $\neg$ E Ψ	A
3	(3)	E Ψ	A
1,3	(4)	E Ω	1,3 EE
1,2,3	(5)	$\neg$ E Ψ	2,4 EE
1,2,3	(6)	E Ψ & $\neg$ E Ψ	3,5 &I
1,2	(7)	$\neg$ E Ψ	3,6 RAA
1,2	(8)	E Ω	2,7 EI
1,2	(9)	E Ψ	1,8 EI
1,2	(10)	E Ψ & $\neg$ E Ψ	7,9 &I
1	(11)	Not: Ω expresses that $\neg$ E Ψ	2,10 RAA
2	(12)	Not: Ψ expresses that E Ω	1,10 RAA

The upshot is that either the assumption of line 1 or the assumption of line 2 cannot represent a truth, leaving open the possibility that both are untrue. This appears disconcerting because one should suppose that, in view of what has been stipulated at the beginning, it is both the case that Ψ expresses that Ω expresses something true, represented by ‘E Ω ’, and that Ω expresses that Ψ fails to express a truth, represented by ‘ $\neg$ E Ψ ’. To be sure, the supposition that some (linguistic) item should ascribe to itself lack of truth or failure of expressing a truth appears unfamiliar. But one can hardly detect any plausible basis for the view that such self-ascriptions are impossible in principle. Be this as it may, the situation envisaged in II*) shows that the existence of apparently disconcerting consequences pertaining to matters of semantics does not essentially depend on the question of the possibility of direct self-reference and self-ascription. Furthermore, it should be noted that the exact import of the word ‘express’ or of any

other term used to designate the pertinent relation does not matter for this argument. One might replace the word ‘express’ by some variable term, say ‘ ϕ s’, and write ‘ $\Psi \phi$ s that $E \Omega$ ’ (instead of ‘ Ψ expresses that $E \Omega$ ’) and ‘ $\Omega \phi$ s that $\neg E \Psi$ ’ (instead of ‘ Ω expresses that $\neg E \Psi$ ’), under the supposition that ‘ $x \phi$ s y ’ designates any appropriate relation holding between (linguistic) units and contents representing truth ascriptions or truth denials: The derivability of a formal contradiction vindicates the impossibility of two items, such that one represents a truth ascription to some (different) item which in its turn represents a truth denial with respect to the former. Here the question whether the word ‘express’ is an appropriate term for signalling the pertinent relation between expressions and contents is not decisive.

C) Inconsistencies implied by apparently consistent suppositions regarding possible arrangements of T-comments

Let us suppose that there is some (actually) infinite series of T-comments, such that each member of the series denies truth with respect to all members subsequent to it. A series of this sort can be represented by various formulations, e.g. by

(a) A1: All subsequent items express an untruth, A2: All subsequent items express an untruth, A3: All subsequent items express an untruth, ...

(a*) A1: All subsequent items fail to express something true, A2: All subsequent items fail to express something true, A3: All subsequent items fail to express something true, ...

(b) B1: No subsequent item is true/expresses a truth, B2: No subsequent item is true/expresses a truth, B3: No subsequent item is true/expresses a truth, ...

(b*) B1: There is no subsequent item which expresses a truth, B2: There is no subsequent item which expresses a truth, B3: There is no subsequent item which expresses a truth, ...

Against this background one can reason as follows:

III)

- (1) Let us assume that any member M of the series, which could be the very first or any subsequent item, expresses a truth.
- (2) Then it must be true that every member subsequent to M is untrue/fails to express a truth.
- (3) Therefore the immediately succeeding member $M+1$, as well as all items following it, are untrue/fail to express a truth.
- (4) Since $M+1$, as well as all following items, express that all following items are untrue/fail to express a truth, it must be untrue that all members subsequent to $M+1$ are untrue.¹⁶
- (5) Consequently there must be some member subsequent to $M+1$ that expresses a truth.
- (6) This result militates against the consequence, formulated in (2), implied by the pertinent assumption, formulated in (1).
- (7) Hence M must be untrue/fail to express a truth.
- (8) Since M expresses that all subsequent members are untrue/fail to express a truth, it must be untrue that all members subsequent to M are untrue/fail to express a truth.¹⁷
- (9) There must be some member $M+N$ following M which expresses a truth.
- (10) The reasoning demonstrating the impossibility of M 's truth can be applied to $M+N$ establishing that both the assumption of M 's truth and M 's absence of truth entail contradictions.

¹⁶ Or alternatively: Since $M+1$, as well as all following items, express that no following item is true/expresses a truth, it must be untrue that no member subsequent to $M+1$ is true/expresses a truth.

¹⁷ Or alternatively: Since M expresses that no subsequent member is true/expresses a truth, it must be untrue that no member subsequent to M is true/expresses a truth.

The case under consideration and the above presented piece of reasoning are based on the so-called ‘Yablo-Paradox’ about which it has been claimed (by its inventor) that it does not involve any self-reference.¹⁸ At this point it is not needed to go into the details of the discussions about that claim. It should suffice to remark that some sort of self-reference is in fact involved inasmuch as the term ‘subsequent’ in (a)–(b*) represents a relational notion equivalent to ‘subsequent to this’, where ‘this’ refers to the item within which it occurs. However, there is no intuitive basis for claiming that III) involves a self-reference of the kind that can be discerned in II) or II*), where a T-comment appears to involve a direct or indirect comment on itself. But one could dispute the relevance of the considered example by maintaining that it militates against the previous requirement that truth paradoxes ought to exhibit the characteristic that apparently unacceptable consequences arise out of possible situations, for it is doubtful that the existence of an (actually) infinite series of linguistic items is possible. It has to be admitted, of course, that the supposition of the occurrence of an infinite sequence of linguistic items does not represent a realistic possibility, according to an intuitive understanding of the expression ‘realistic possibility’. But one should not go so far as to deny that the envisaged possibility can ‘in principle’ be realized, for the pertinent assumption does not seem to involve an inconsistency. Nevertheless, this defence will not meet with universal assent, for it can be maintained that the concept of an ‘actual infinity’ is flawed in some other way than by being inconsistent.

One must indeed admit that the same paradoxical consequences would not arise if the situation of a *finite* series of subsequent truth denials (of any length) were considered — so that one might distinguish between ‘infinite Yablo sequences’ and ‘finite Yablo sequences’. If one orientates oneself by the formulations employed in (b) and (b*) this is obvious: The very last member of any finite Yablo sequence should express a truth so that any previous member can be relegated to the realm of untruths. As far as (a) and (a*) are concerned,

¹⁸ Cf. Yablo 1993:251–252 as well as Clark 2012:267ff [2012:292ff].

ordinary intuitions might waver with respect to the question whether one ought to classify a statement referring to all subsequent members as true or not, if no such members exist. Anyhow, if truth were denied one might consistently attribute truth to the penultimate member and failure of expressing a truth to the preceding members.¹⁹ Against this background it is important to note that similar predicaments as in cases of infinite Yablo sequences can be discerned by considering slight modifications of finite Yablo sequences, such as:

C1: All subsequent items fail to express a truth, C2: All subsequent items fail to express a truth, C3: All subsequent items fail to express a truth, C4: The immediately preceding item expresses a truth.

A pertinent piece of reasoning might proceed as follows:

IV)

- (1) Let us suppose that C1 expresses a truth.
- (2) Then C2, C3 and C4 fail to express a truth.
- (3) Then C3 must express a truth because it would express a truth about C4. Thus a contradiction results.
- (4) Let us instead assume that C1 fails to express a truth. Then C2, C3 or C4 must express a truth.
- (5) If C2 expresses a truth then both C3 and C4 must fail to express a truth. But then C4 must express a truth so that an inconsistency results.
- (6) If C3 expresses a truth then C4 must fail to express a truth, so that C3 fails to express a truth and an inconsistency results.
- (7) If C4 expresses a truth then C3 expresses a truth so that C4 must fail to express a truth and an inconsistency arises again.
- (8) Accordingly, C1 cannot fail to express a truth either.

¹⁹ The circumstance that, by hypothesis, all members exhibit the same formulation does not refute this possibility because the identity of linguistic content does not necessitate the identity of the type of content which is relevant for truth evaluation. (More about this will be said later.)

Obviously the state of affairs would not be substantially altered if additional items exhibiting the same formulation as C3 were inserted immediately before the ultimate member of the series. The crucial factor is that the relationship between C3 and C4 is exactly like that between Ω and Ψ in Π^*). Thus self-reference plays an important role in the considered modification of finite Yablo sequences. But the predicament that both assumptions of truth and of absence of truth lead to inconsistencies here affects even elements which do not occur as (direct or indirect) objects of T-comments, such as C1.

These observations cannot refute the thesis that all invocations of an infinity of related T-comments entail special problems. For the time being, this issue need not be pursued and can be taken up later. What matters here and now is that in the preceding expositions of pieces of reasoning both the principle represented by '[P1]', '(TE1)' or 'EE', and the principle represented by '[P2]', '(TE2)' and 'EI' come into play. The first one supports, for example, the steps from (1) to (2) in III) and in IV). And the reasoning in (4) and (8) within III) as well as that in (3) and (4) within IV) appeals to the second principle. For the theorem:

If s expresses that p then (if p then $E s$)

is equivalent to:

If s expresses that p then (if $\neg E s$ then $\neg p$).

D) Derivability of contingent (historical) propositions from assumptions solely relating to non-contingent matters

Let us suppose that ' Γ ' and ' Δ ' designate respectively the expressions:

' $E \Delta$ '

and (with ' $p \rightarrow q$ ' for 'if p then q ')
' $E \Gamma \rightarrow$ Napoleon Bonaparte was an orthodox Hindu'

This amounts to a conditional attribution of truth on the part of one of the expressions, viz. ' Δ ', to another expression, viz. ' Γ ', saying that if it, viz. ' Γ ', expresses a truth, then Napoleon Bonaparte was an

orthodox Hindu. The same effect could be achieved by a sequence of two sentences, such that the first reads:

The (immediately) subsequent sentence expresses a truth.

and the second:

If the (immediately) preceding sentence expresses a truth, then Napoleon Bonaparte was an orthodox Hindu.

Let us use 'N' as an abbreviation for 'Napoleon Bonaparte was an orthodox Hindu'. Applied to the pertinent example the principles (TE1) and (TE2) run as follows.

With respect to Γ :

If Γ expresses that $E \Delta \ \& \ E \Gamma$, then $E \Delta$

If Γ expresses that $E \Delta \ \& \ E \Delta$, then $E \Gamma$

and with respect to Δ :

If Δ expresses that $[E \Gamma \rightarrow N] \ \& \ E \Delta$, then $E \Gamma \rightarrow N$

If Δ expresses that $[E \Gamma \rightarrow N] \ \& \ E \Gamma \rightarrow N$, then $E \Delta$

In the following arrangement of derivational steps those states-of-affairs will be introduced as assumptions and the sequence can be presented as follows:

V)

1	(1)	$(\Gamma \text{ expresses that } E \Delta \ \& \ E \Gamma) \rightarrow E \Delta$	A
2	(2)	$(\Gamma \text{ expresses that } E \Delta \ \& \ E \Delta) \rightarrow E \Gamma$	A
3	(3)	$(\Delta \text{ expresses that } [E \Gamma \rightarrow N] \ \& \ E \Delta) \rightarrow (E \Gamma \rightarrow N)$	A
4	(4)	$(\Delta \text{ expresses that } [E \Gamma \rightarrow N] \ \& \ (E \Gamma \rightarrow N)) \rightarrow E \Delta$	A
5	(5)	$\Gamma \text{ expresses that } E \Delta$	A
6	(6)	$\Delta \text{ expresses that } [E \Gamma \rightarrow N]$	A
7	(7)	$E \Gamma$	A
5,7	(8)	$\Gamma \text{ expresses that } E \Delta \ \& \ E \Gamma$	5,7 &I
1,5,7	(9)	$E \Delta$	1,8 MPP
1,5,6,7	(10)	$\Delta \text{ expresses that } [E \Gamma \rightarrow N] \ \& \ E \Delta$	6,9 &I
1,3,5,6,7	(11)	$E \Gamma \rightarrow N$	3,10 MPP

1,3,5,6,7	(12) N	7,11 MPP
1,3,5,6	(13) $E \Gamma \rightarrow N$	7,12 CP
1,3,5,6	(14) Δ expresses that $[E \Gamma \rightarrow N] \ \& \ (E \Gamma \rightarrow N)$	6,13 &I
1,3,4,5,6	(15) $E \Delta$	4,14 MPP
1,3,4,5,6	(16) Γ expresses that $E \Delta \ \& \ E \Delta$	5,15 &I
1,2,3,4,5,6	(17) $E \Gamma$	2,16 MPP
1,2,3,4,5,6	(18) N	13,17 MPP

One could, but need not, continue as follows:

1,2,3,4,5,6	(19) $E \Delta \rightarrow N$	15,18 CP
1,3,5,6,7	(20) $\neg N$	A
1,3,5,6	(21) $\neg E \Gamma$	13,20 MTT
1,3,5,6	(22) $\neg E \Delta$	19,20 MTT
1,3,4,5,6	(23) $E \Gamma \ \& \ \neg E \Gamma$	17,21 &I
1,3,4,5,6	(24) $E \Delta \ \& \ \neg E \Delta$	15,22 &I
1,2,3,4,5,6	(25) Not: Δ expresses that $[E \Gamma \rightarrow N]$	6,23/24 RAA
1,2,3,4,5,6	(26) Not: Γ expresses that $E \Delta$	5,23/24 RAA ²⁰

The final outcome of the extended version of V) closely resembles that of II), and in particular II*): It is impossible to satisfy all stipulations concerning the contents expressed by Γ and Δ . The result depends, however, on the contingent circumstance, that the state of affairs described by ‘N’, viz. that Napoleon Bonaparte was an orthodox Hindu, does not obtain. In contrast, the final conclusion of the non-extended version, formulated in line (18), solely depends on the original assumptions in lines (1)–(6). The thought that a historically untrue proposition should be derivable from those assumptions is perplexing. For the first line stipulates that if the item designated by ‘ Γ ’ expresses (or means) that the item designated by ‘ Δ ’ expresses a truth, then it holds true that if the item designated by ‘ Γ ’ in fact expresses a truth, then the term designated by ‘ Δ ’ also expresses a truth.²¹ Similarly the

²⁰ ‘MPP’, ‘CP’ and ‘MTT’ designate the rules of ‘*modus ponendo ponens*’, ‘conditional proof’ and ‘*modus tollendo tollens*’ respectively.

²¹ This paraphrase is legitimate because ‘ $(p \ \& \ q) \rightarrow r$ ’ is equivalent to ‘ $p \rightarrow (q \rightarrow r)$ ’.

third line stipulates that if the item designated by ‘ Δ ’ expresses that if the item designated by ‘ Γ ’ expresses a truth then Napoleon Bonaparte was an orthodox Hindu, then it must hold true that if the item designated by ‘ Δ ’ expresses a truth then it must be the case that if the item designated by ‘ Γ ’ expresses a truth then Napoleon Bonaparte was an orthodox Hindu. This amounts to an application of the idea that if some (linguistic) item L expresses that p , then it holds good that if L (thereby) expresses a truth, then it is the case that p . On the other hand, the lines 2 and 4 represent applications of the idea that if something expresses something or other, then it holds good that if what is thereby expressed is a truth then the item expressing it expresses a truth. Since those two ideas appear like truisms, at least as long as items that are free of ambiguities are concerned, it is hard to believe that they should constitute a basis for deriving false contingent propositions. As the subsequent lines (5) and (6) merely represent the pertinent stipulations concerning the expressed content (or the meaning) of the items designated by ‘ Γ ’ and ‘ Δ ’, it appears perplexing that they in combination with the remaining assumption of (1)–(4) enable the derivation of an untrue proposition concerning contingent historical matters. But that is not all: One can easily see that by the method of V(1)–(18) it would be possible to derive *any* arbitrary (true or untrue) proposition. One would only have to replace ‘ N ’ by some other item — or bestow on the letter ‘ N ’ a different interpretation than before. Instead of employing ‘EE’ and ‘EI’ as derivation rules, the applications of the underlying principles have been introduced as assumptions in lines (1)–(4), for in this manner it becomes particularly conspicuous that the possible derivation of arbitrary propositions needs to rely only on most elementary derivation rules, specifically ‘&I’ and ‘MPP’.

A derivation possessing a similar effect has been presented in Künne 2013:55–57 against the background of the assumption that

some Cretan says or asserts that all Cretans assert nothing but untruths. The sequence of derivational steps is as follows²²:

VI)

1	(1)	$K[\forall x (Kx \rightarrow Fx)]$	A
2	(2)	$W[\forall x (Kx \rightarrow Fx)]$	A
2	(3)	$\forall x (Kx \rightarrow Fx)$	2; W-elimination
2	(4)	$K[\forall x (Kx \rightarrow Fx)] \rightarrow F[\forall x (Kx \rightarrow Fx)]$	3; $\forall$ -elimination
1,2	(5)	$F[\forall x (Kx \rightarrow Fx)]$	1,4; MPP
1	(6)	$W[\forall x (Kx \rightarrow Fx)] \rightarrow F[\forall x (Kx \rightarrow Fx)]$	2,5; CP ²³
1	(7)	$W[\forall x (Kx \rightarrow Fx)] \rightarrow \neg W[\forall x (Kx \rightarrow Fx)]$	6; substitution $\neg$ -W/F
1	(8)	$\neg W[\forall x (Kx \rightarrow Fx)]$	7; Consequentia Mirabilis
9	(9)	$W[\forall x (Kx \rightarrow Fx)] \vee F[\forall x (Kx \rightarrow Fx)]$	A
1,9	(10)	$F[\forall x (Kx \rightarrow Fx)]$	8,9; Disjunctive Syllogism
9	(11)	$K[\forall x (Kx \rightarrow Fx)] \rightarrow F[\forall x (Kx \rightarrow Fx)]$	1,10; CP

The intended interpretation of the symbol ‘K’ is ‘is asserted by a Cretan’, ‘W’ and ‘F’ abbreviate the German counterparts of ‘is true’ and ‘is false’ (or ‘is not true’) respectively, and square brackets surrounding a sentence are used to form a singular term designating what is said by that sentence. Now according to (10), the assertion of a Cretan to the effect that Cretans assert only untruths cannot be true, given (1) that such an assertion was actually made and (9) that what was asserted is either true or false (not true). So, it has to be granted that, if the event of a Cretan’s attributing untruth to all assertions made by Cretans actually occurred, there must be some *different* assertion made by a Cretan which is not false (untrue). Although it is highly probable that on a different occasion some Cretan has indeed asserted something else that was true, the idea that this contingent

²² Künne’s entries on the right-hand side of lines (8) and (10) invoke theorems of the propositional calculus in order to abbreviate the derivation. The theorem appealed to in (8) is ‘ $(p \rightarrow \neg p) \rightarrow \neg p$ ’, the theorem appealed to in (9) is ‘ $((p \vee q) \& \neg p) \rightarrow q$ ’.

²³ Since it is line (2) that contains the discharged assumption, I took the liberty of replacing Künne’s entry ‘1,5; $\leftrightarrow$ Einführung (= CP)’ by ‘2,5; CP’.

fact should be guaranteed just by the circumstance that an (untrue) assertion of the described kind has been made, appears very astonishing.

In some respect the paradox generated by VI) is weaker than the puzzles which can be distilled from V), for they demonstrate that any arbitrary true or untrue proposition can be derived relying only on basic rules of derivation.

E) Imposed partiality of truth-value assignments

Let us consider the following argument:

VII)

1	(1)	H expresses that $\neg E \Theta$	A
2	(2)	Θ expresses that $\neg E H$	A
3	(3)	$E H$	A
1,3	(4)	$\neg E \Theta$	1,3 EE
5	(5)	$\neg E \Theta$	A
1,5	(6)	$E H$	1,5 EI
7	(7)	$E \Theta$	A
2,7	(8)	$\neg E H$	2,7 EE
9	(9)	$\neg E H$	A
1,9	(10)	$E \Theta$	2,9 EI
1	(11)	$E H \rightarrow \neg E \Theta$	3,4 CP
1	(12)	$\neg E \Theta \rightarrow E H$	5,6 CP
2	(13)	$E \Theta \rightarrow \neg E H$	7,8 CP
2	(14)	$\neg E H \rightarrow E \Theta$	9,10 CP
1	(15)	$(E H \rightarrow \neg E \Theta) \& (\neg E \Theta \rightarrow E H)$	11,12 &I
2	(16)	$(E \Theta \rightarrow \neg E H) \& (\neg E H \rightarrow E \Theta)$	13,14 &I
1,2	(17)	$(E H \leftrightarrow \neg E \Theta) \& (E \Theta \leftrightarrow \neg E H)$	15,16 &I ²⁴

The point can be illustrated by the following example.

²⁴ ‘ ’ for ‘if and only if’. We have abstained from presenting a slightly shorter argument that applies the rule of ‘*modus tollendo tollens*’.

Let us imagine a situation where two persons A and B are colleagues who participate in some conference. Let us further suppose that the sentences

(U1) Not a single truth is said by my colleague B during this conference.

(U2) Not a single truth is said by my colleague A during this conference.

are assertively uttered by A and by B respectively and that those utterances constitute the only contributions of A and B to the pertinent conference.²⁵ Surely, these assumptions depict a possible situation. It is easy to verify that, at least according to ordinary intuitions concerning truth and saying something true, it is both impossible that both utterances express a truth and that both utterances express an untruth.²⁶ In accordance with (TE1) truth of (U1) implies untruth of (U2) and vice versa. An assumption of untruth with respect to one of them would imply the truth of the other one according to (TE2). The only consistent ways of attributing truth appear either an assignment of truth to (U1) and of lack of truth to (U2) or, the other way around, an attribution of truth to (U2) and of untruth to (U1). That in itself is not inconsistent. Consistency could be maintained even with respect to two utterances of a sentence obtained from (U1) and (U2) by deleting the names 'B' and 'A' respectively, since the occurrence of the indexical element ('my') allows for differences of (propositional) content. The crux lies rather in the fact that opting for one of the two opposing truth-assignments would be utterly arbitrary. It would be of no value to appeal to possible differences of reputation concerning the speakers of (U1) and (U2), for attributions of truth or

²⁵ Although the assumption that (during the conference) neither A nor B says anything else *which is true* would suffice for the subsequent exposition, the suggestion that (U1) and (U2) are the only contributions made by the two colleagues is proposed in order to underscore the similarity between the members of the pair.

²⁶ Here 'express a truth (an untruth)' stands for the more cumbersome formulation 'are used to say something that is true (not true)'.

untruth to individual statements should in the final analysis depend on what is said and not on the question who said it.²⁷ Is there a way of avoiding unfounded partiality and of substantiating the judgement that (U1) and (U2) deserve exactly the same verdict?

Quite similar observations can be made if one considers an infinite series of truth denials, where each member disputes truth with respect to its immediately subsequent member. If the initial member expressed a truth the immediately following member must fail to express a truth. But then — if one accepts (TE2) — the third member would express a truth, and so on. Apparently, however, no basis exists for preferring this way of assigning truth values against the alternative, where lack of truth is assigned to the initial member, possession of truth to the second, absence of truth to the third, and so on. If the series were finite the situation would be different: In accordance with the evaluation of the final member truth-assignments of the preceding items could be determined. To be sure, the situation would change again if the last member expressed a negative comment on a preceding element of the series.

It might appear tempting to solve the dilemma by maintaining that truth of a T-comment requires the existence of some item which is not a T-comment and which constitutes either a direct or at least an indirect object of the pertinent T-comment, as illustrated by:

(T)

T0: T1 does not express a truth.

T1: Napoleon Bonaparte made war on Prussia.

(T*)

T0: T1 does not express a truth.

T1: T2 expresses a truth.

T2: T3 expresses a truth.

T3: Napoleon Bonaparte made war on Prussia.

²⁷ Even those who think otherwise would be led into an impasse by the (not unrealistic) assumption that the speakers of (U1) and (U2) deserve the same degree of respect or disrespect.

But for various reasons one should be reluctant to adopt this ‘solution’:

1. One would have to assume that in a finite sequence like

(T**)

T0: The following remark is true.

T1: No subsequent item expressing a truth occurs.²⁸

T0 could not be assessed as true, although intuitions support the view that here both T0 and T1 express something that is true.

2. In a sequence such as

(T***)

T0: The subsequent remark is true.

T1: The preceding sentence either expresses a truth or fails to do so.

it would be more in accordance with ordinary intuitions to attribute truth to T0 than to consider it as untrue — or as neither true nor false.

3. With respect to a sequence like

(T****)

T0: The subsequent remark expresses a truth.

T1: If the preceding sentence expresses a truth then Napoleon Bonaparte made war on Prussia.

— which is reminiscent of the situation envisaged in V) — intuitions do not license a sweeping rejection of the supposition that T0 expresses something true. It appears even more counterintuitive to deny T0 any assertible content. It seems, after all, that T0 can serve as a basis for derivations.

4. Let us grant for the sake of argument that both (U1) and (U2) above lack truth or even fail to assert anything. But aren’t we impelled to admit that precisely under such circumstances both (U1) and (U2) tells it the way it really is?

These considerations should yield a sufficient motivation for seeking after alternative solutions.

²⁸ Or alternatively: ‘If there is any subsequent item, then it does not express a truth’.

III. Truth of T-comments

It is not difficult to ascertain that a great number of different arguments and considerations engendering paradoxes rely on exploitations of the principles (TE1) and (TE2) — or of variants that essentially come to the same thing as these principles. (TE1), viz.

[For any linguistic unit s :] If s expresses that p and s thereby expresses a truth then p

appears incontrovertible, and the same holds for every suitable substitution instance of the schema

If s ϕ s that p and s thereby ϕ s a truth, then p .

Since something can be related in the same way to different items, it may be apposite to distinguish a weaker and a stronger reading of the conjunct ‘ s expresses a truth’ in the antecedent of (TE1). According to the weaker reading it amounts to the same as

s expresses something which is a truth.

According to the stronger reading the phrase is equivalent to

s expresses nothing but a truth

or more explicitly

s expresses something which is a truth and there is nothing expressed by s which is not a truth.

In the formulation of (TE1) that was initially presented the expression ‘thereby’ was inserted under the supposition that the weaker reading might be attributed to ‘ s expresses a truth’. If, however, the stronger reading is assumed the addition is dispensable. As mentioned earlier, the difference between the alternative readings becomes immaterial whenever it can be supposed that an item is not simultaneously related to different contents in the way signalled by the term ‘expresses’ (or

similar ones). Hence, if duly explicated, (TE1), at least as long as the linguistic unit *s* does not contain indexical elements, appears incontrovertible.

As far as (TE2), viz.

[For any linguistic unit *s*:] If ((*s* expresses that *p*) and *p*), then *s* thereby expresses a truth.

is concerned, the constituent ‘thereby’ in the consequent clause is required only if, with respect to this clause, the stronger reading is envisaged and could be omitted otherwise. Even here the difference can be neglected if one does not have to account for the possibility that a pertinent item expresses different propositions. Thus the phenomenon of ambiguity does not suffice to discredit the validity of (TE1) and (TE2) but calls at most for suitable explications. Even if one would speak in a less explicit manner of truth and lack of truth with respect to *sentences* — instead of truth or lack of truth of something expressed or conveyed etc. by linguistic items — the difference can be accounted for by expressions like ‘is true under some reading’, ‘is true under all (admissible) readings’, ‘is untrue/fails to be true under some reading’, ‘is untrue/fails to be true in under all (admissible) readings’ or — to account for the possibility that some item is devoid of sense and admissible readings — ‘does not possess any reading under which it is true’.

Given that (TE1) and (TE2) come into play in a great variety of considerations and derivations leading to or involving paradoxes and that (TE1) appears most incontrovertible, the conjecture suggests itself that in the final analysis (TE2) — as well as [P2] and the derivation rule ‘EI’ — have to be abandoned. From a merely formal viewpoint a derivation of the untruth of specific instances of (TE2) and the non-validity of (TE2) as a general principle could be easily accomplished. For example, instead of adding in argument V) after line (23) and (24) the final lines (25) and (26), conveying that at least one of the pertinent expressions cannot express that which they were supposed to express, one could add:

- 1,3,4,5,6,20 (25*) Not: $((\Gamma \text{ expresses that } E \Delta) \& E \Delta)$
 $\rightarrow E \Gamma$ 2,23/24 RAA
- 1,2,3,5,6,20 (26*) Not: $((\Delta \text{ expresses that } [E \Gamma \rightarrow N])$
 $\& [E \Gamma \rightarrow N]) \rightarrow E \Delta$ 4,23/24 RAA

This amounts to claiming that if one a) maintains that the concerned items in fact express that which they were supposed to express, b) accepts the validity of (TE1) with respect to the pertinent case and c) takes it that Napoleon Bonaparte was in fact not an orthodox Hindu, then at least one of the instantiations of (TE2) must fail to be true. Similarly, by a minor transformation of I), which reads:

I')

- | | | | |
|---|-----|---|---------|
| 1 | (1) | $E \Delta$ | A |
| 2 | (2) | $\neg E \Delta \rightarrow E \Delta$ | A |
| 1 | (3) | $\neg E \Delta$ | 1 EE |
| 1 | (4) | $E \Delta \& \neg E \Delta$ | 1,3 &I |
| | (5) | $\neg E \Delta$ | 1,3 RAA |
| 2 | (6) | $E \Delta$ | 2,5 MPP |
| 2 | (7) | $E \Delta \& \neg E \Delta$ | 5,6 &I |
| | (8) | Not: $\neg E \Delta \rightarrow E \Delta$ | 2,7 RAA |

it would convey that the instance of (TE2) that is represented in line (2) must be unconditionally rejected. Analogous arrangements could be made in II), II*) and in other cases.

At this point one faces an objection based on familiar considerations. It runs as follows:

It is absolutely illegitimate to contest (TE2). For compliance with the antecedent clause in (TE2) means that things really are as we, by using the pertinent (linguistic) item, say they are, or that things are as the item in question says they are. But this can only mean that the concerned linguistic entity expresses a truth.²⁹

Although the considered suspension of (TE2) need not entail a global rejection of its validity but merely an abrogation in special cases, it is

²⁹ Cf. also Künne 2003:334.

apposite to raise the general question of what it may mean that things really are as some linguistic unit, in particular a declarative sentence, says they are and that it accordingly expresses a truth. Let us first examine the issue by considering a sentence which is not a T-comment. Suppose that 'II' designates the English sentence:

'Napoleon Bonaparte made war on Prussia'

The question as to what it means, in the envisaged sense of the word, that things are as II says they are and that accordingly II expresses a truth, suggests the following answer. It means at least two things, namely:

(a) that the English sentence 'Napoleon Bonaparte made war on Prussia' (in fact) expresses a truth

and

(b) that it is a truth that Napoleon Bonaparte made war on Prussia.

In this context 'that p means that q' amounts to the same as 'it cannot be the case that p unless it is the case that q'. The component (a) arises as a most immediate consequence because, trivially, something can only be the case if it is the case. (b) on the other hand must also be regarded as an indispensable requirement for the truth of II: If it should be the case that (a) is satisfied it must equally hold good that something exists that is expressed by II and which is a truth or which is true; given however, that the proposition that Napoleon Bonaparte made war on Prussia is the only conceivable content which can be rated as something that is linguistically expressed by II, the truth of this content needs to constitute an indispensable condition for expressing a truth on the part of II. The conjunction of (a) and (b) represents a necessary, and presumably sufficient, condition for II's expressing a truth.

An application of the same principle to Λ in I) yields the following result:

Λ 's expressing a truth means, that is, requires

(a) that Λ in fact expresses a truth

and

(b) that (it is a truth that) Λ fails to express a truth,

hence it requires the satisfaction of a contradiction, provided that Λ expresses precisely a denial of the proposition specified in (a). Since Λ does involve such a denial it can be deduced that the content specified in (b) should not establish Λ 's truth; as it is the content which Λ expresses, Λ implies even a denial of this content. This in its turn suggests that someone voicing a T-comment attributing truth to Λ endorses the tenet that it is in fact possible that certain linguistic items express a truth and fail to do so at the same time.

Analogous consequences can be ascertained with respect to Σ in II). Regarding the situation considered in II*) the corresponding outcome is that, given the validity of (TE1), it is necessary for the truth of Ψ that

(α) Ψ expresses a truth

(β) Ω expresses a truth

(γ) Ψ fails to express a truth.

Deeply entrenched intuitions apparently militate against the supposition that those three conditions might be satisfied together. But if the validity of (TE2) were not acknowledged with respect to Ω , (α) could be still regarded as a requirement for the truth of Ψ , but the obligation to derive (α) from (β) or the untruth of (β) from (γ) would become otiose. This circumstance suggests that attributing lack of truth or failure of expressing a truth to Ψ does not necessarily lead to a contradiction. It involves an inconsistency merely in combination with either the assumption that Ψ actually expresses a truth or the supposition that (TE2) possesses unrestricted validity.

Our intuitive reluctance to attribute truth to either Λ , Σ or Ψ can be traced back to an easily comprehensible basis: The assumptions that

Λ , Σ or Ψ expresses a truth and that (a), (α) or an analogous proposition concerning Σ is true implies a contradiction which can be derived from the pertinent assumptions by bringing (TE1) but not (TE2) into play. This fact can be made conspicuous by invoking a derivation rule of ‘*self-entailment*’ (‘SE’ for short) permitting to repeat any line of an argument, such that the repeated line depends on exactly the same assumptions as its predecessor. It implements the idea that every proposition entails itself. Logically, the rule is dispensable, of course, but in the present connection it serves the purpose of enhancing the transparency of our representation of paradox-ridden situations. Employing SE one can replace II) — and accordingly other pieces of reasoning — by the following derivational sequence:

(IIA)

1	(1)	Σ expresses that $\neg E \Sigma$	A
2	(2)	$E \Sigma$	A
2	(3)	$E \Sigma$	2 SE
1,2	(4)	$\neg E \Sigma$	1,3 EE
1,2	(5)	$E \Sigma \ \& \ \neg E \Sigma$	3,4 &I
1	(6)	$\neg E \Sigma$	2,5 RAA

Thus, it becomes fully manifest that under certain circumstances an acknowledgment of (TE2) leads to consequences which blatantly contradict propositions which are derivable by employing most plausible principles — provided the property of being highly plausible is acknowledged not only with respect to the idea that every proposition entails itself but also regarding (TE1) and the corresponding rule ‘EE’. By admitting the validity of ‘SE’ and ‘EE’ and admitting in addition that propositions which make contradictions derivable can never be true one is compelled to recognize that it must be impossible that items like Σ express anything true. This evidently raises the problem of explaining why it ought to be impossible to attribute (expressing a) truth to the concerned units despite the fact that apparently the content which they express is not contradictory or inconsistent.

The subsequent attempt at an explanation proceeds in two steps.

(1) In the light of the previous observation that the truth of Π requires that the conditions specified in both (a) and (b) are fulfilled it appears imperative to suppose that the truth of a T-comment ascribing truth to that item requires, apart from the circumstance that it expresses a truth itself, first, that Π in fact expresses a truth and second, that in fact Napoleon Bonaparte made war on Prussia. Hence a full explication of the necessary requirements of expressing a truth on the part of a T-comment ascribing truth to Π would involve more than that which the comment explicitly expresses — namely the fact that Π expresses a truth — even if one leaves out of account the requirement that it expresses a truth itself. Against this backdrop we can differentiate between two sorts of content which might be labelled (i) ‘*linguistic(ally conveyed) content*’ and (ii) ‘*holistic content*’. With regard to Π , the first type of content would correspond to the proposition that Π expresses a truth whereas the holistic content consists in the conjunction of the propositions that Π expresses a truth and that Napoleon Bonaparte made war on Prussia, i.e. all the relevant truth-requirements except the condition that the comment itself expresses a truth. The difference between these two varieties of content can be generalized and applied to all kinds of T-comment. It would be natural to identify the holistic content of a T-comment denying truth with respect to Π with the conjunction of the propositions that Π expresses a truth and that it is not the case that Napoleon Bonaparte made war on Prussia, so the truth denial would entail the falsity of the content that is (linguistically) expressed by the item that functions as an object of comment. But it is important to note that the situation need not always be like this: Generally speaking, a truth denial claims merely that *not all conditions which would be required for something’s being true are satisfied*. Therefore a truth disclaimer could be rendered true either by the circumstance that some unit functioning as an object of comment does not possess any intelligible meaning permitting to associate the item with a definite content or by the fact that the item lacks any content which is capable of being true or untrue or by the untruth of some relevant content that need not be equivalent to the content which the object of comment linguistically expresses.

(2) The combination of (TE1) and (TE2) amounts to the same as a principle which can be expressed by the formulation

(TE*) [For any linguistic unit *s*:] If *s* expresses that *p*, then *s* (thereby) expresses a truth *if and only if p*.

It can be gathered from this formulation that the combination of (TE1) and (TE2) implies some specification regarding the pertinent meaning of the word ‘express’. If the validity of (TE*) is acknowledged, it follows that the verb ‘express’ is to be employed in such a manner that whenever some item expresses something or other it must hold good that the item expresses a truth (in the pertinent sense of the term ‘express’) if and only if that which it expresses is the case. Considered in isolation the above specified theorem could be regarded as a mere stipulation which could compete with alternative stipulations, in particular with one implied by a principle that reads

(TE1*) [For any linguistic unit *s*:] If *s* expresses that *p*, then *s* (thereby) expresses a truth *only if p*.

It is not difficult to discern that the alternative represented by (TE1*) is induced by the supposition that (TE1), but not (TE2) possesses unrestricted validity. But it seems that a propensity to admit the validity of (TE*) is widely shared, hence a disposition to employ the terms ‘express’ and particularly ‘express a truth’ — or similar terms — in a sense that complies with that principle should be quite common. This disposition is clearly discernible in pieces of reasoning alluded to before which argue that if an item claiming its own untruth were in fact untrue, it would follow that it expresses a truth and is true — because, allegedly, things would stand exactly as the item says they stand.

As a matter of fact, one can reasonably dispute the proposition that if sentences disclaiming truth regarding themselves fail to be true it inevitably follows that things stand exactly as those sentences say they stand. If the import of the truth denials articulated in Λ and Σ conforms with (TE*) then, according to an important and natural sense of ‘say’, they do not say everything which is relevant for their expressing something true. The formulation of a T-comment ascribing the quality

of (expressing a) truth to Π typically does not reveal that (if it expresses a truth then) it is a truth that Napoleon Bonaparte has made war on Prussia, in other words, that Π expresses a truth *by expressing that Napoleon Bonaparte made war on Prussia*. A truth denial formulated by

Π is not true / Π does not express a truth / What Π expresses is not a truth.

does not 'say' in the envisaged sense of the word, that (if the comment is true then) Π cannot express something true *on account of the fact that it expresses that Napoleon Bonaparte made war on Prussia*. In the same way Λ and Σ do not explicitly convey that (if they were true then) they cannot express a truth by expressing that they do not express anything true. Specifically under the considered assumption concerning the import of the term 'express', they do not (explicitly) say that their expressing something true does not only require that they fail to express a truth but also that the proposition that they fail to express a truth is not a truth. What they explicitly convey represents merely a necessary condition but not a condition that is both necessary and sufficient for their (expressing a) truth. A full specification of their relevant truth requirements is only provided by the *holistic* content which is the conjunction of the propositions that the concerned sentences do not express a truth and that it is not a truth that those sentences do not express a truth. Similarly, if the denial expressed by Ω in Π^* conforms with (TE*), then the item fails to say, in the pertinent sense of this word, that the proposition that Ω expresses a truth is not true. Accordingly, the *holistic* import of Ω is to be identified with the contention that Ψ fails to express a truth *and* that it is not true that it itself, i.e. Ω , expresses something true.

On the other hand, the consequences would palpably differ, if the truth disclaimers were to possess an import that is not in conformity with (TE*) but rather with (TE1*). If their import were to comply with (TE1*) the supposition that Λ or Σ fail to be true would not license the derivation of the consequence that it is not the case that the pertinent items fail to express a truth. The relevant supposition would merely

amount to the claim that it is not the case that all the requirements for expressing a truth on the part of Λ or Σ are met, hence there would be no commitment to the effect that lack of truth is due to the untruth of the explicitly conveyed content. As regards Ω in Π^* , its truth-denial regarding Ψ would imply that it cannot be the case that all necessary truth requirements regarding Ψ are satisfied; but there would be no (strict) entailment to the effect that the truth ascription expressed in Ψ is an untruth. The variant conceptions of being related to a content correlate with differences of holistic import. Truth denials which are in accordance with (TE^*) represent stronger claims than truth denials complying with $(TE1^*)$. Accordingly, as far as the items considered above are concerned, truth paradoxes are generated exclusively on the basis of concepts of content-expression that accord with (TE^*) .

Although linguistically conveyed and holistic contents of T-comments generally differ, they often, and presumably in a vast majority of cases, correlate with equivalent propositions. If a T-comment attributes truth to Π it will express a truth not only if and only if Π in fact expresses a truth, but also if and only if it is a fact that Napoleon Bonaparte made war on Prussia. Similarly, a denial of truth regarding Π ought to express a truth precisely if Π fails to express a truth and if it is not a truth that Napoleon Bonaparte made war on Prussia. This circumstance could explain the existence of an entrenched disposition to conceive the word 'express' or similar terms connecting linguistic units with contents as being in accordance with (TE^*) and thus to interpret truth denials as implying negations of a content explicitly conveyed by the commented item(s). But the fact that cases exist where the equivalence does not hold good does not only demand a differentiation between the two varieties of import but also calls for a terminological distinction between corresponding types of relations connecting linguistic items with content. In this vein let us introduce a terminological distinction between '*linguistically express*' or simply '*express*' for relations between expressions and linguistic contents and '*holistically express*' or '*express**' for relations between expressions and holistic contents. A parallel difference pertaining to the notion of meaning can be signalled by terminologically distinguishing between

'*mean*' and '*mean**'. With respect to T-comments '*mean*' serves to signify a relation between a T-comment and an import induced by its linguistic meaning, while '*mean**' is employed to signify a relation between a T-comment and *all* the relevant requirements for its (expressing a) truth. In accordance with these stipulations, one is entitled to assert that a T-comment characterising Π as a sentence that expresses a truth means that Π is a sentence that expresses a truth and means* that Π is a sentence that expresses a truth and that Napoleon Bonaparte made war on Prussia. Generally, a sentence of the form '*X means* that p*' can be paraphrased by formulations such as '*X's being true/expressing a truth means that p*' or '*that X is true/expresses a truth means that p*'. Against this background it becomes patent that strictly speaking the relationship signalled by '*mean**' does not directly pertain to linguistic items and to propositional contents but rather to qualities or facts pertaining to linguistic items on the one hand and to propositions on the other. Even T-comments generating truth paradoxes do not refute the contention that those propositions are qualified for the status of bearers of absolute truth.

Only holistic contents can provide exhaustive and explicit specifications of conditions relevant for the truth of T-comments. Provided that holistic contents amount to conditions that are not only necessary but also sufficient for truth, the relational concept signified by '*express**' complies with (TE*). If the existence of the circumstance that (it is the case that) *p* is necessary and sufficient for the existence of the circumstance that some item *S* expresses a truth so that it holds good that

S expresses a truth if and only if (it is the case that) *p*.

then the principle

[For any linguistic unit *s*:] If *s* expresses* that *p*, then *s* (thereby) expresses* a truth if and only if *p*.

ought to be generally valid under the proviso that the holistic contents which constitute the range of the relation of expressing* provide necessary and sufficient conditions of truth for items in the domain.

Hence it would be totally inappropriate to dismiss (TE*) and (TE2) as invalid. Instead one needs to add a qualification to the effect that they are valid *with respect to holistic contents*, but not, at least not generally, with respect to contents linguistically conveyed in T-comments. The root of the paradoxes of truth considered so far lies in the circumstance that the principles (TE*) and (TE2) are associated with inappropriate types of content. The prejudice that a failure to express a truth on the part of a negative T-comment must be due to the untruth of the explicitly conveyed content of its object of comment is apt to induce the disconcerting consequence that the pertinent T-comment can neither be assessed as true nor as failing to be true. The urge to accept this consequence vanishes if the pertinent presupposition is dropped by acknowledging the possibility that truth-failure of a T-comment can result from the untruth of its holistic content which alone represents its necessary and sufficient conditions of truth, conditions that are not fully manifested by its formulation.

Against the background of the different varieties of content as well as different concepts of expressing a content it is not difficult to explain how one can consistently attribute absence of truth or failure of expressing a truth to the items Λ , Σ and Ψ in I), II) and II*). One (presumably) asserts something true if one contends that those items do not express* something that is true because, if the concepts of expressing referred to in those sentences comply with (TE*) they entail — in a way that does not depend on the validity of (TE*) — that the concerned units both fail to express something true and do not fail to express something true. Provided one accepts that contradictions cannot be true and that assumptions entailing contradictions cannot be true either, one is obliged to admit that everybody who expresses a denial of the truth of the holistic contents connected with Λ , Σ and Ψ thereby asserts a truth. The crucial reason lies in the circumstance that somebody who disclaims the truth of the pertinent holistic contents makes a different and weaker claim than anybody who claims in addition that the failure of expressing a truth on the part of the relevant items must be due to the untruth of their linguistically conveyed imports. The upshot is that the holistic contents of T-comments commenting on T-comments

can differ in accordance with different conceptions of expressing a content, even if the difference need not (always) be manifested on the linguistic level. This is the case if a terminology is employed which does not reflect the distinction between 'express' and 'express*' as explained above. In this manner differences of truth value correlate with differences of (holistic) content.

A most important consequence of the account delineated in this section is that assessing matters of truth and specifying conditions of expressing truth essentially transcend the framework of linguistic or philological investigations. They require, at least from a theoretical perspective, to bring a concept of meaning into play which differs from the conception of meaning that stands in the focus within linguistics and traditional philology. If this phenomenon were restricted to the realm of few specimens of truth denials it might be dismissed as practically irrelevant. So, the circumstance that paradoxes of truth are not confined to T-comments expressing or implying truth denials regarding themselves possesses significance.

IV. Applications of the Theory

Although (TE2) plays a pivotal role for the derivation of paradoxes a rejection of (TE2) by itself falls short of providing satisfactory answers to various questions our attempt to (dis)solve truth paradoxes is likely to provoke. The following (nine) applications of that account are meant to shed more light on the nature of that account by addressing worries which may still persist.

(1)

One might find it puzzling that a restriction of validity should be imposed on (TE2) which does not affect in the same manner (TE1). According to the account delineated above (TE1) is even applicable to kinds of content which are explicitly conveyed by linguistic units for deriving consequences from the assumption that they express a truth. Does this amount to an undue prejudice favouring (TE1) over (TE*) and (TE2)? As a matter of fact, the adopted position does not involve any unjustified partiality. (TE2) clearly represents a potentially stronger claim than (TE1), and this potential becomes specifically significant in the realm of T-comments. The reason why (TE2) involves a stronger claim than (TE1) lies in the circumstance that a statement to the effect that

If (it is the case that) p then s by expressing that p expresses a truth.
is potentially more vulnerable than

If s by expressing that p expresses a truth, then (it is the case that) p.

If the condition that a sentence expresses a truth comprises among others the truth of the (propositional) content which is explicitly expressed by it, then the pertinent condition, i.e. the sentence's expressing a truth, is potentially stronger than the truth of the explicitly

expressed proposition; the former entails the latter, but not necessarily vice versa. In (TE1) the stronger requirement appears in the antecedent and the weaker in the consequent clause, whereas in (TE2), the other way around, the weaker in the antecedent and the stronger in the consequent clause.³⁰ Hence, other things being equal, a satisfaction of (TE1) guarantees a satisfaction of (TE2), but not necessarily vice versa. To be sure, in cases such as Π truth of the explicitly expressed content constitutes both a necessary and a sufficient condition of the truth of the item expressing that content.³¹ But as soon as one takes T-comments into account the situation fundamentally changes. The reason is that T-comments can, and usually, do involve an implicit (!) comment pertaining to the content(s) linguistically conveyed by the items functioning as their objects of comment and that its correctness constitutes a necessary requirement for the T-comment's (expressing a) truth.³²

³⁰ The circumstances that (TE2) is equivalent to the principle that, if a linguistic item expressing that p fails to express a truth then (it is) not (true that) p , puts into relief a crucial aspect of its vulnerability: If it were universally valid, untruth of expressed content would be the only possible reason for failure of expressing a truth. However, in I) and other specimens of derivation the consequence of failure of expressing a truth is not based on the untruth of the expressed content of a pertinent unit but on the fact that the supposition of its expressing a truth leads to an inconsistency (if TE1 is not repudiated).

³¹ Considering (unequivocal) linguistic items containing indexical constituents one ought to say more precisely: The truth of the proposition determined by the explicitly expressed linguistic meaning in combination with pertinent contextual features constitutes a necessary *and* sufficient condition for the sentence's expressing a truth.

³² The property of implicitly taking a position on contents associated with (other) items is not confined to the area of T-comments. If a speaker A were to utter assertively the

Nothing which is (or has been) asserted by me has ever been asserted by B.
the assertion can only be true — given that the universal quantification is not restricted — if, in fact, B has never asserted that nothing asserted by A has ever been asserted by him, and if it should happen that both A and B have once asserted that the earth is round, the pertinent assertion cannot be true (at least according to one reading of the sentence).

(2)

The last contention is compatible with the fact that T-comments do not always imply substantial verdicts about the content of their objects, a fact which comes to light if one considers examples like

Napoleon Bonaparte was an orthodox Hindu. That sentence either expresses a truth or fails to do so.

But examples like the one considered in V) indicate that also in the realm of complex T-comments implicit comments on qualities of objects of comment are by no means generally irrelevant. A major reason lies in the circumstance that (natural) languages permit the coinage of linguistic units which exhibit the peculiarity that their lack of truth warrants the truth of their explicitly expressed content. In order to cast more light on the pertinent matter it appears suitable to consider an informal piece of reasoning based on a modification of the example considered in V). It exhibits an immediate instead of an indirect self-reference and is reminiscent of the so-called ‘Curry-paradox’ — or ‘Löb’s paradox’. Let us suppose that some speaker U utters

(Z) If the sentence which I am uttering just now expresses a truth then Napoleon Bonaparte was an orthodox Hindu.

one could reason as follows:

VIII)

- (1) Suppose that the sentence (Z) uttered by U expresses a truth.
- (2) Then (it is true that) if the sentence (Z) uttered by U expresses a truth then Napoleon Bonaparte was an orthodox Hindu [in accordance with (TE1)].
- (3) Consequently, it holds true that if the sentence (Z) uttered by U expresses a truth then Napoleon Bonaparte was an orthodox Hindu [as this conclusion is vindicated by the rule of Conditional Proof it does not depend on the original assumption].

(4) But (Z) expresses (in the supposed context) exactly this, namely that if the sentence (Z) uttered by U expresses a truth then Napoleon Bonaparte was an orthodox Hindu.

(5) Consequently, the sentence (Z) uttered by U expresses a truth [in accordance with (TE2)].

(6) Ergo: Napoleon Bonaparte was an orthodox Hindu [by *modus ponendo ponens* from 3 and 5].

If this piece of reasoning were impeccable, cases would exist where an utterance of a linguistic item of the form ‘If A then B’ validates the conclusion that the antecedent of the conditional expresses a truth, where the truth of this conclusion does not depend on the assumption that the antecedent expresses in fact a truth. But the derivation of line (5) from (4) is not possible if (TE2) does not automatically license the conclusion that an item can be considered as being true or as expressing a truth given that its explicitly expressed content represents a truth and if one insists that (TE2) can be applied only if the complete set of conditions of being true, which comprise, among others, the (a)-component, viz. that the concerned item is in fact true or expresses a truth, is fulfilled. If (TE2) can be applied only under the more restricted conditions³³ the conclusion of line (6) could not be vindicated by the circumstance alone that (it is true that) if the sentence (Z) uttered by U expresses a truth then Napoleon Bonaparte was an orthodox Hindu but only by this circumstance *in combination with* the supposition that the sentence (Z) uttered by U actually expresses a truth. This means that the assumption formulated in the first line constitutes an indispensable factor for the truth of the proposition expressed in line (6). Thus at least in the realm of T-comments the assumption that its (a)-component holds true can constitute a substan-

³³ This means that (TE2) is applicable with respect to the content which is explicitly expressed by the sentence: ‘The sentence (Z) uttered by U expresses a truth and if the sentence (Z) uttered by U expresses a truth then Napoleon Bonaparte was an orthodox Hindu.’

tial addition to the supposition that its (b)-component, viz. its explicitly formulated content, is a truth. In the case of (Z) lack of truth of the (a)-component should guarantee the truth of the (b)-component, given that a (material) conditional ‘If A then B’ expresses a truth whenever the antecedent clause fails to express a truth. Seen in this light the thesis that truth of the (b)-component cannot by itself guarantee the truth of the (a)-component and that accordingly the supposition of the truth of the latter can constitute a substantial addition to the supposition of the truth of the former harmonizes with a pre-theoretic intuition.

Since

- (i) the application of (TE2) — in combination with (TE1) — to an explicitly expressed content has the effect that its truth must constitute not only a necessary but also a sufficient condition for the ascription of (the property of expressing a) truth to an item expressing it, and
- (ii) with respect to specific T-comments, such as (Z), the requirements for truth ascriptions are thereby relaxed in a way that even lack of truth licenses them,

propositions are derivable which could otherwise not be derived. In the case of examples like (Z) the crucial factor lies in the derivability of a proposition which occurs as an antecedent in an equally derivable conditional creating the condition for the applicability of the rule of *modus ponendo ponens*. It emerges as a result that expressing a truth on the part of (Z) would require the truth of the proposition that Napoleon Bonaparte was an orthodox Hindu, whereas this is not necessary for the truth of the explicitly expressed propositional content.

Quite similar observations can be made regarding pieces of reasoning which start from assumptions concerning the co-referentiality of different linguistic items. In this connection certain fallacies based on inappropriate basic assumptions have to be forestalled. In particular formulations of the forms

- (i) $A = 'A \rightarrow B'$
- (ii) $A = (A \rightarrow B)$
- (iii) $A \leftrightarrow (A \rightarrow B)$ or: A if and only if $(A \rightarrow B)$

are unsuitable. The first two have to be avoided on account of syntactic and semantic incongruities: The identity symbol '=' requires that the symbol 'A' at its left represents a singular term, whereas the occurrence of the same symbol at the left of ' $\rightarrow$ ' requires that it represents a sentence.³⁴ (iii) is syntactically well-formed but represents an invalid formula, which becomes obvious if one replaces 'A' by a truth and 'B' by an untruth; it had been laid down, however, that the term 'truth paradox' ought to be applied only in situations where (apparently) unacceptable conclusions are derivable from *prima facie* plausible assumptions. The reason for mentioning those facts lies in the circumstance that they seem to be unduly disregarded now and then.³⁵

In contrast the following formulation

- (iv) $A = 'E A \rightarrow B'$

where 'E' is to be interpreted as before, is acceptable and can serve as a basis for the following argument:

IX)

1	(1)	$A = 'E A \rightarrow B'$ ³⁶	A
2	(2)	$E A$	A
1,2	(3)	$E 'E A \rightarrow B'$	On account of identity-assumption in (1)
1,2	(4)	$E A \rightarrow B$	Based on (TE1)
1,2	(5)	B	2,4 MPP
1	(6)	$E A \rightarrow B$	2,5 CP

³⁴ In order to avoid unneeded complications it is preferable not to present the matter in a way which suggests the disputable Fregean conception of sentences as names (of truth values).

³⁵ Cf. In this connection the expositions of Curry's paradox in the 'Wikipedia' or Bocheński/Menne 1973:92–93.

³⁶ Obviously, in this formula 'A' is a placeholder for singular terms, while 'B' is a placeholder for sentences.

- | | | |
|---|-----------------------------|--|
| 1 | (7) E 'E A $\rightarrow$ B' | Based on (TE2) |
| 1 | (8) E A | On account of identity-assumption in (1) |
| 1 | (9) B | 6,8 MPP |

According to (TE2), if something is the case then anything which expresses that this is the case expresses a truth. Here again this principle is exploited for deriving some arbitrary conclusion. It turns out that the truth of B is a necessary requirement for accrediting truth to A — which is clearly manifested by line (5), as it depends (apart from the assumption of the first line) on line (2) — although it is not necessary for the truth of the explicitly expressed content of the sentence designated by A.

The situation envisaged in IX) represents a realistic possibility. One can, for example, specify contextual features to an extent that only one linguistic item is produced in a particular context. Given that 'C' signifies such a unique context of utterance and that in C the sentence

If the (only) sentence uttered in C expresses a truth, then Napoleon Bonaparte was an orthodox Hindu.

is uttered: Then the expressions 'the (only) sentence uttered in C' and "If the (only) sentence uttered in C expresses a truth, then Napoleon Bonaparte was an orthodox Hindu" denote the same linguistic item (token or type), so that

the (only) sentence uttered in C = 'if the (only) sentence uttered in C expresses a truth, then Napoleon Bonaparte was an orthodox Hindu.'

can be assumed.

(3)

The circumstance that in all these cases a T-comment's being true definitely relies on additional factors apart from the truth of its explicit content, inasmuch as its truth (indirectly) depends on the content and truth values of items which function as objects of comment, furnishes additional support for disputing the general validity of (TE2). At this

point one has to face the following objection: The claim that the advocated thesis harmonizes with common intuitions can hardly be maintained. For isn't it plain that a T-comment expresses something true whenever that which it explicitly expresses is true? Here one has to take into account an equivocation inhering in expressions, such as 'express a truth', 'express something which is true', which is not considered in the previously highlighted distinction between different readings of those phrases coping with the problem that items functioning as objects of T-comments can be ambiguous. If one considers a complex sentence like

Managua is the capital of Nicaragua and Napoleon Bonaparte was born in Austria.

in some sense it would be legitimate to say that the pertinent item expresses a truth or something which is true. The reason lies evidently in the fact that the first component of the conjunction is, or expresses, a truth. But it is equally legitimate, of course, to deny ascription of truth to the complex expression because one conjunct is not true. For the time being it is not necessary to dispute the contention that in the former and weaker sense of the phrase 'express a truth' even T-comments like Λ , Σ , Ψ , Ω , Γ , Δ etc. might express something which is true. In any case, ascriptions of truth to T-comments should be orientated by the principle that the truth of all conjuncts in a conjunction is required for ascriptions of truth to a conjunction as a whole. There is no need to embark on discussions about the question which of the variant interpretations harmonizes better with common linguistic intuitions. What matters in the present connection is rather the following: 1) The proposed interpretation must not militate against common linguistic practice, 2) it must be clear and intelligible, 3) it ought to yield theoretical benefits, 4) it should not be favoured only for the sake of allowing for some *ad hoc* solution of theoretical problems, 5) the views built on this stance ought to possess a considerable degree of plausibility even if one disregards the fact that truth paradoxes exist.

(4)

It could be preferable to abstain from the employment of the term ‘express’ for signalling a relation between meaningful linguistic items and something that transcends their explicitly represented content. The previously considered terminological distinctions between ‘express’ and ‘express*’ as well as between ‘mean’ and ‘mean*’ embody a possible way of overcoming relevant defects. Against this background one could with respect to formulations representing T-comments in a less explicit manner employ the ideas of being true or being untrue (failing to be true) *under an interpretation*. Expressions like ‘S expresses a truth’, ‘S fails to express a truth’ exhibit by themselves a particular sort of vagueness resulting from the circumstance that the ordinary term ‘express’ is underdetermined: It can be explicated in a way that it (exclusively) concerns the explicitly expressed linguistic contents of units which function as object(s) of comment; but it can be equally explicated as a relation between linguistic items and ‘holistic’ contents that are not fully manifested by their linguistic import. Consequently, it can occur that T-comments are alternatively classifiable as being true, i.e. as expressing a truth, or as untrue, i.e. as failing to express a truth, in accordance with the way in which they are interpreted and explicated. This phenomenon will now be elucidated in more detail.

The linguistic content of (Z), for example, viz.

If the sentence which I am uttering just now expresses a truth then Napoleon Bonaparte was an orthodox Hindu.

consists in the conditional that if the sentence (Z) uttered by U expresses a truth then Napoleon Bonaparte was an orthodox Hindu, whereas its holistic counterpart encompasses the conjunction of the linguistic content together with the proposition that Napoleon Bonaparte was an orthodox Hindu.³⁷ This shows how substantial the

³⁷ The explication

If the sentence which I am uttering just now expresses a truth then Napoleon Bonaparte was an orthodox Hindu, and if that is a truth — i.e. if it holds good that if the sentence which I am uttering just now expresses a truth then

disparity between the two content-types can become. But it is a common phenomenon that truth-value ascriptions do not explicitly indicate a contrast between different types of content. To be sure, regarding sentences of the form ‘It is (not) true that p’ as for instance

(S1) It is (not) true that Napoleon Bonaparte was an orthodox Hindu.

it holds usually good that the truth of their explicitly expressed content constitutes a necessary as well as a sufficient condition for their expressing a truth. But truth-assessments of T-comments, such as

(S2) That is (not) true.

require first an explication which specifies the contextually induced or intended reference of the deictic expression, which could be provided by, say:

(S2*) That, namely (the proposition) that Napoleon Bonaparte was an orthodox Hindu, is (not) true.

In such cases one might distinguish between (i) a linguistically expressed content in the strict sense and (ii) a propositional content induced by the linguistic content. Considering examples, such as:

(S3) If what our president has said is true, then two plus two equals ten million.

one can equally ascertain that evaluating truth or lack of truth can only be accomplished if what was said by the contextually pertinent president is specified, as, for example, in the following explication:

Napoleon Bonaparte was an orthodox Hindu — then Napoleon Bonaparte was an orthodox Hindu.

exhibiting the form of ‘If p then q, and if (if p then q) then q’ which equivalent with ‘If p then q, and q’ reveals this fact.

(S3*) If (what our president has said, namely) that the situation in our country is better than ever before is true, then two plus two equals ten million.³⁸

Here one could argue that the truth of the explicitly expressed content need not matter for the question of whether one ought to agree or disagree with the statement. Agreement or disagreement need not be affected by the issue of whether it was in fact *said* by a president that the situation in some particular country is better than ever before. Decisive importance might be rather attributed to the question of whether or not it is really the case that the situation in the pertinent country is better than ever before. But this presumably only shows that one needs to differentiate between the question of a T-comment's expressing a truth on the one hand and the legitimacy of attributing truth to the utterance made by its employment on the other. Moreover, it is easy to see that apart from a literally expressed content both (S3) and (S3*) suggest an implicature to the effect that it is completely absurd to suppose that the situation (at some particular time) in some country is better than ever before. It emerges that apart from linguistic and holistic contents also other types of contents can function as a basis for truth assessments concerning T-comments.

The possibility of substantial divergences between linguistic and holistic contents arises because T-comments typically possess implications pertaining to truth or absence of truth of contents that are not specified by their literal import. This can happen, as in the examples (S2) and (S3), because a T-comment, although referring to some content, does not present a specification of the content to which it refers. Similarly as in the case of deictic expressions or definite descriptions like 'The favourite colour of Hans Dietrich Genscher' where one can appropriately ask 'But what is the favourite colour of Hans Dietrich Genscher?' one can ask analogous questions with respect to (S2) and (S3): 'But what is that which is allegedly true or

³⁸ A well-grounded judgment about truth or lack of truth would require further specifications as regards the reference of 'our country' and as regards the time which functions as the implicit relatum of the word 'before'.

not true?', 'What is that which the president said?'. The same predicament occurs if a T-comment does not contain elements purporting to refer to any content at all. Most pertinent examples are items referring to linguistic units, specifically individual sentences, as well as T-comments exhibiting quantifications, such as

- (S4) The last sentence on page four does not express a truth.
- (S5) All sentences expressing universal propositions are untrue.
- (S6) All assertions Nancy made about Napoleon are untenable.
- (S7) Nothing true is ever said on this spot.

They all imply consequences pertaining to the possession or lack of truth of possible propositional contents that are not specified by them.³⁹ In this regard it does not matter whether they refer to linguistic or non-linguistic entities or whether they do not refer to individual objects at all. Since sentences referring to linguistic objects share with other sorts of T-comments the highlighted crucial feature, focussing for the sake of illustration on cases which exhibit reference to linguistic items does not distort the account. A basis for differentiating between different types of content, and in particular between linguistic and holistic content, can be detected even if one disregards truth paradoxes.

The truth of a negative T-comment, i.e. of a truth denial, which claims that not all necessary requirements for (expressing a) truth are fulfilled, depends on various factors. This idea appears most natural against the background of the existence of a larger plurality of contents. One can in fact establish that there are several possibilities, such as 1) meaninglessness of items functioning as objects of T-comments, 2) non-existence of (propositional) content, e.g. in the case

³⁹ Even with respect to sentences implying truth denials regarding their own linguistic content like

The sentence which I am uttering just now does not express any truth at all.

one can ascertain that the implication about their own content, viz. that it is not a truth that the sentence the speaker is uttering does not express a truth at all, is not, and cannot be, (linguistically) expressed by the sentence the speaker is uttering.

of interjections, which can be credited with meaning but not with a content assessable as true or untrue, 3) untruth of some literally expressed content, 4) untruth of a propositional content induced by literal meaning in combination with features of context, if linguistic tokens are at stake, 5) untruth of some holistic content, 6) untruth of implicatures induced by ‘conversational maxims’. In the present connection the varieties 3)–5) possess primary importance. The circumstance that those differences are not signalled by common formulations must not mislead one into supposing that they are irrelevant. On the contrary, this fact is suited to highlight the significance of explications.

(5)

In section III above it was shown by argument II.A) that an item Σ that supposedly expresses that it does not express something true entails a contradiction, given (i) that all instances of ‘ $P \rightarrow P$ ’ are true and that the corresponding derivation rule of ‘self-entailment’ is valid and (ii) that (TE1) holds true and the corresponding derivation rule ‘EE’ is valid. It was suggested that this fact should motivate us to reject any supposition to the effect that Σ expresses a truth. Now this might appear counterintuitive for the following reason: Assuming it is in fact true that Σ fails to express a truth and that Σ linguistically expresses that Σ fails to express a truth; should one not admit that that which Σ linguistically expresses is a truth and that accordingly there must be something true which Σ linguistically expresses? Should we not reject the supposition that Σ ’s entailing a contradiction provides a solid basis for denying Σ ’s truth? Or should we admit in the end that Σ is both true and untrue?

In the light of the foregoing observation the following reply might appear natural: The supposition that Σ is both true and fails to be true is merely apparently absurd; the two formulations ‘ Σ is true’ and ‘ Σ fails to be true’ only evoke the impression of incompatibility because they are not sufficiently explicit – which is due to the absence of any

reference to a relation between linguistic items and content. Nevertheless, mere reference to a relation cannot provide a satisfying solution because the two formulations ‘ Σ expresses a truth’ and ‘ Σ fails to express a truth’ evoke the impression of incompatibility to the same degree. But it may appear that our distinction between ‘express’ and ‘express*’ offers a way-out. Could one not maintain that Σ expresses a truth and (at the same time) fails to express* a truth? It is important to recognize that and why this proposal cannot provide an appropriate answer to everything which matters in this area.

First, it cannot be taken for granted that the item designated by ‘ Σ ’ (linguistically) expresses a definite content capable of being categorically true. In the previous discussion the concept of expression that is a constituent of the import of Σ was not definitely fixed. This leaves room for the possibility that the pertinent notion of content expression is vague, being indeterminate between the alternatives of linguistically or holistically expressing. If, however, the tenet that Σ expresses a truth and fails to express* a truth were in fact correct it would be illegitimate to suppose that the circumstance that it linguistically expresses a truth could categorically bestow truth on the item, precisely because of the indeterminacy of its import. One could at most claim that in the current circumstances the item can be assessed as true on one explicatory reading and as failing to be true on another reading.

Second, and this is presumably more important, even on the supposition that the terminological framework in which Σ is embedded exhibits a degree of elaborateness that is (at least) as high as the one in which its assessment is embedded and that it refers to a definite concept of linguistic expression, the contention that ‘there must be something true which Σ linguistically expresses’ is less plausible than it initially appears. The structure of the linguistic import, as it has been conceived, can be explicated by the formulation

For every x : If x is a content linguistically expressed by Σ then x is not a truth.

Now let us suppose that it is really the case that there is something true which Σ linguistically expresses. Then, given that no restriction is

imposed on the universal quantification, the content of Σ patently militates against the pertinent supposition. Σ entails that if anything is a truth then this truth cannot be expressed by it. Thus, Σ denies the applicability of (TE2) to itself. Let 'c' stand for 'nothing which Σ linguistically expresses is a truth/is true'. Then the formulation

For every x: If x is a content linguistically expressed by Σ , then x is not a truth, and if Σ linguistically expresses (the content) that c, then (the content) that c is not a truth/is not true.

yields an explication of the character of the pertinent universal quantification. In accordance with the principle ' $p \rightarrow q \vdash (p \& r) \rightarrow q$ ' the second conjunct, viz. 'and if ...', implies

If Σ linguistically expresses (the content) that c and it is true that c, then (the content) that c is not a truth which is linguistically expressed by Σ , i.e. Σ does not express a truth by linguistically expressing that nothing which Σ linguistically expresses is a truth.

It becomes apparent that and why one can consistently maintain (i) that it is a truth or a fact that Σ does not express any truth and that this fact can be asserted in principle and (ii) that this truth cannot be expressed as a truth by Σ itself. The reason why one can truthfully assert the proposition that Σ does not linguistically express any truth by means of a different linguistic item, namely 'Nothing which Σ linguistically expresses is a truth', lies in the circumstance that by using that formulation one does not in the same manner counteract the truth of what is linguistically expressed and that one does not (implicitly) dispute the applicability of (TE2) to that content. It is Σ 's way of implicitly counteracting which is apt to bestow truth on a T-comment denying Σ 's truth.⁴⁰

Obviously, a linguistic item's possessing a content does not imply its possessing a content that is capable of being assessed as true. To be sure, the manner in which Σ has been introduced assures us that the

⁴⁰ One might compare this with a situation where somebody utters

It is true that Napoleon Bonaparte made war on Prussia, and when(ever) I say that Napoleon Bonaparte made war on Prussia I do not say anything true.

item designated by ‘ Σ ’ is a meaningful unit and on that account it should be credited with the quality of expressing something or other. It is, nonetheless, easy to ascertain that in this sense of expressing or possessing a content many linguistic items which in virtue of their meaning alone are not related to propositional contents exhibit this property. Examples are

- The first Nobel Prize laureate in physics
- is a Nobel Prize laureate
- Since W. C. Röntgen is the first Nobel Prize laureate in physics
- W. C. Röntgen is the first Nobel Prize laureate in physics and

The decisive point is that even from the circumstance that some unit belongs to the category of meaningful and complete declarative sentences it cannot be deduced that the pertinent item expresses a propositional content capable of being assessed as true, given that certain states of affairs obtain.⁴¹ The suspicion that Σ belongs to a category of (linguistically impeccable) sentences whose content is not assessable as true is fuelled by the consideration that its content is specified on the basis of the notion of its own (expressed) content. As soon as one raises the question:

What is (the content) expressed by Σ ?

and finds oneself confronted with the answer:

The content expressed by Σ is that Σ does not express anything which is true.

⁴¹ Thus, from the assumption that Σ does not express any truth (whatsoever) one cannot derive, by the rule of universal quantifier elimination (or universal instantiation or specification) the conclusion that it is not a truth that Σ does not express any truth — and by the rules of ‘&’-introduction and *reductio ad absurdum* that Σ must express some truth. It can be only legitimate to infer that the content that Σ does not express any truth is not a truth which is expressed by Σ , i.e. that it is not both expressed by Σ and a truth. Only at this point the ordinary conception of expressing something — which orientates itself by linguistic items not involving self-reference or self-comment — requires an adjustment.

one is inevitably thrown back to the question:

What does it mean* that Σ does not express anything which is true? and at this stage it appears to be impossible to specify any non-circular explanation of some condition which would be sufficient for vindicating its truth.⁴² In this regard it does not matter, whether the term 'express' is to be explicated by 'linguistically express' or 'holistically express' or in some other way. Being confronted with the grammatically well-formed (English) sentence:

This is true.

one can easily discern (provided that one possesses the required linguistic competence and even supposing that a token of this sentence occurs as an example in a linguistic treatise) that it (the type or a token) expresses the same as

Das ist wahr.

in German or

Det är sant.

in Swedish etc. Here it is evident that knowing what something means does not automatically involve knowledge as to what something means*.

This observation is compatible with the fact that sometimes truth-conditions which are not question begging can be specified for items exhibiting this feature. Even disregarding questions of the interpretation of the letter 'A', 'B' or 'C' one can ascertain that, say,

$A \rightarrow B$

must represent a truth if 'B' is assessable as true or that

$(A \ \& \ B) \ \& \ C$

⁴² The situation clearly contrasts with the state of affairs that can be ascertained, for example, regarding a truth denial concerning the item designated by 'N' in the context of V) in section II.D above.

can only be true if 'C' is assessable as true. For *this* question not only the issue regarding the values of 'A' and 'B' can be discounted but even the query of whether 'C' stands i) for an item whose content would be assessable as true given alternative circumstances or ii) for an entity that lacks any such content. On such a basis it is claimed that Σ needs to exhibit absence of truth, given that (TE1) and the impossibility of the truth of contradictions ought to be acknowledged.

Absence of linguistically expressed content capable of being assessed as true does not rule out the possession of a content which together with other contents is able to constitute some content that is definitely assessable with respect to truth or lack of truth. It is equally compatible with the suppositions that the (allegedly non-propositional) linguistic content of Σ can function as a target of (quasi-)reference and constitute an object of comment. This possibility is vindicated, e.g. by:

Paul has won the first prize and this is something of which everybody can be proud.

Eric is something which many politicians are and this is something of which one should be ashamed.

Here the occurrences of 'this' do not refer to objects like persons, events or propositions but substitute expressions like 'winning the first prize' and 'to be something which many politicians are'.⁴³

In this manner a difficulty is circumvented that emerges in a most conspicuous manner as soon as one considers a slightly modified wording of Σ which reads:

⁴³ From a technical point of view the imminent dilemma might be dissolved by stipulating that generally the linguistic contents of declarative sentences, even if they do not contain indexical constituents, are not propositions but functions of the type 'P and' so that such sentences express propositions only in combination with certain circumstantial conditions, such as the circumstance that a linguistic item does not imply a T-comment on itself or on its own content. — Note that line (2) in argument II) in section II or in argument II.A) in section III does not presuppose that Σ exhibits a content that is capable of being true. The introduction of this assumption merely conveys what would be true if the item were to possess a true content.

(Σ^+) What Σ^+ expresses is not true / is not a truth.

Let us hypothesize that the following identity holds good:

What Σ^+ expresses = the proposition that what Σ^+ expresses is not true / is not a truth.

Then the subsequent consequences have to be accepted:

What Σ^+ expresses is true if and only if the proposition that what Σ^+ expresses is not true is *true*.

What Σ^+ expresses is untrue if and only if the proposition that what Σ^+ expresses is not true is *untrue*.

After all, if

The first Nobel Prize laureate in physics = Wilhelm Conrad Röntgen

holds true it must be accepted as well that

The first Nobel Prize laureate in physics was born in Lennep if and only if Wilhelm Conrad Röntgen was born in Lennep.

etc. But if the identity hypothesized above is not maintained the disconcerting consequences do not result. The view that the expression 'what Σ^+ expresses' does not refer to any specific proposition is corroborated by the consideration that any attempt to specify the import of the item designated by it would in its turn involve a reference to the notion of being something which Σ^+ expresses.

The factor that bestows truth on the proposition that Σ fails to express a truth is a 'formal' circumstance, namely the fact that the supposition of its (expressing a) truth entails a contradiction and that it cannot be true that Σ , or any other linguistic item, fails to express a truth and at the same time does not fail to do so. It appears indeed incongruous to suppose that the factor which alone renders untrue the complex holistic content and imparts truth on its denial should equally bestow truth, or respectively untruth, to its elementary components.

One could say that only the holistic content represents a complete sense = thought.⁴⁴

Given that the holistic content of Σ amounts to a contradiction⁴⁵ and assuming that not linguistic but holistic contents deserve to be treated as a basis for truth-value assignments — and that linguistic contents can play the same role only in a (admittedly) great number of cases where the different types of content do not materially differ — it follows that the pertinent item can be definitely assessed as being untrue or, more precisely, as failing to express any truth. Hence no ground exists for calling into question the validity of the rule of *reductio ad absurdum*. This mirrors the intuitive view that if someone would utter a sentence like

(S7) The sentence which I am uttering just now does not express any truth at all.

he would commit himself, by assertively uttering this sentence, to acknowledge that any (linguistically) expressed content of his own sentence is not a truth and to admit that it is not a truth that the sentence uttered by him does not express any truth.

⁴⁴ If a linguistic item L1 says about a linguistic item L2 that L2 does not express a truth, the content of L1 is incomplete in so far as its being true or untrue depends on what L2 means. If L1 says about L2 that L2 expresses a truth and fails to do so, then the question of L1's truth does not any more depend in the same manner on the meaning and sense of L2. Since the incompleteness characterises T-comments in general, it holds also good if L1 comments on itself, i.e. if L1 = L2.

⁴⁵ A suitable paraphrase would read:

Σ fails to express a truth and that, namely that Σ fails to express a truth, is not a truth/is not true.

Theoretically, this form of paraphrase could be even provided for 'ordinary' sentences, not implying any form of self-comment, for example:

Napoleon Bonaparte made war on Prussia and that, namely that Bonaparte made war on Prussia, is a truth / is true.

for a sentence expressing that Napoleon Bonaparte made war on Prussia. The difference is that in ordinary cases paraphrases of this type do not provide additional information relevant for truth-value assessments.

At this point an objection ought to be forestalled which runs as follows: If Σ should fail to express a truth or can be definitely assessed as lacking the property of being true, then it should be possible to correctly assert that this is the case. Accordingly, it must be possible to express a truth by saying:

Σ fails to express a truth.

But if we are able to express a truth by ascribing absence of truth with respect to Σ , why should Σ fail to express anything true if it ascribes the same attribute to itself? Given that both Σ and some Σ' consist in the same chain of words employed in exactly the same sense, how could only one of them expresses a truth?

A partial response to this objection has been indicated before. In addition one should recall the fact that in the realm of items containing indexical elements linguistic synonymy generally does not go hand in hand with expression of contents possessing identical truth values. Specifically, regarding certain sentences with token-reflexive ingredients, such as

The sentence written here is a well-formed English sentence.

replacements by linguistically synonymous expressions of other languages are even bound to alter the truth value. By assuming that T-comments possessing identical linguistic import can differ with respect to truth value, a phenomenon observable in one realm is merely extended to another realm.

It is, however, also imperative to consider more precisely the question of what one in fact asserts about Σ if one truly asserts that Σ fails to express a truth. Evidently the verdict that Σ fails to express a truth was based on the consideration that its holistic content cannot be true, because both the requirements that the concerned item does not express any truth and that it is not the case that it does not express any truth cannot be satisfied together. But by claiming that Σ entails a contradiction and supposing that the conjunctive propositions

Σ expresses a truth and it is not the case that Σ expresses a truth.

It is not true that Σ expresses a truth and it is not true that it is not true that Σ expresses a truth.

must fail to be true one does not pass a verdict on the truth or lack of truth of either conjunct. One neither disputes the truth of the proposition that Σ expresses a truth in particular nor the truth of the proposition that it is not true that Σ expresses a truth in particular. But the verdict to the effect that Σ fails to express a truth because its holistic content amounts to a contradiction which cannot be true rests on the assumption that Σ itself involves a truth denial with respect to its own linguistically expressed content. Hence the holistic contents of the two T-comments, i.e. Σ and the allegedly true verdict about Σ , markedly differ. In this manner the possibility of assigning different truth values can be traced back to a difference of (holistic) content, and the principle that one and the same (completely specified) proposition cannot possess several mutually exclusive truth values need not be violated. It holds true, however, that certain pairs of T-comments, such as Σ and one expressing a verdict about Σ , which can be used for stating different propositions deviating with respect to truth value, are linguistically synonymous. Hence it is, strictly speaking, not appropriate to say that Σ and the item conveying a verdict about Σ express different propositions; one ought rather to resort to a term like ‘express*’, as explicated above, for formulating the relevant fact. The upshot is that the use of a sentence exhibiting the reading of Σ does not make *mandatory* an interpretation according to which it involves a denial regarding its linguistically conveyed content.

According to the view propagated in the present essay the circumstance that one can assert a truth by using an item expressing that Σ fails to be true or to express anything true is apt to induce a delusion: the delusion that a content consisting in a truth denial regarding some linguistic item exhibits the quality of being true. In reality, however, contents (linguistically) expressed by T-comments ascribing or denying truth regarding linguistic expressions are themselves *never* capable of being categorically true. They cannot exhibit this property as long as they are not supplemented by some specification of a

content which items functioning as their object(s) possess. This is, however, perfectly compatible with the fact that T-comments lacking explicit specifications in this respect can be employed for making true *assertions* pertaining to issues of truth regarding linguistic items against the background of contents which those items possess.⁴⁶ In this light one can discern an affinity between contents expressed by T-comments and contents linguistically conveyed by sentences containing indexical elements.

(6)

The phenomenon that linguistically synonymous items, even if they do not contain deictic ingredients, are able to alternatively convey different propositions potentially deviating with respect to truth value is by no means uncommon in the realm of T-comments. A most elementary instance is provided by T-comments on linguistically ambiguous items, such as

(S8) In every Swedish village there is more than one bank.

The truth value of a T-comment on a token of (S8) is crucially affected by the identity of the (linguistic) content of that token.

Similar situations can even occur if items functioning as objects of T-comments are free from linguistic ambiguities in the genuine sense of the term. A pertinent example is provided by sentences whose quantifications can be, so to speak, ‘literally’ or ‘pragmatically’ interpreted as illustrated by

(S9) All visitors must buy a parking ticket.

Such a statement can naturally be interpreted in a sense which would be explicated by:

(S9*) All visitors who enter by car must buy a parking ticket.

⁴⁶ Given that one accepts that true assertions can be made by truth denials commenting on items that do not possess any propositional content one should rather say: ‘against the background of contents which those items possess or the fact that they do not possess any propositional content’.

According to a, presumably less natural, but not impossible interpretation the universal quantification possesses a strict import so that the sentence should be rather explicated by the paraphrase:

(S9**) All visitors, no matter how they enter, must buy a parking ticket.

A comment on (S9) can pertain alternatively to the content expressed by (S9*) or to the one expressed by (S9**) without presupposing a linguistic ambiguity in the strict sense of this term. Examples like these do not only reveal how easy it is to irritate other people by wilfully imputing imports which were not intended, perhaps not even conceived before as possibilities. They also disclose that sometimes little is achieved by considering exclusively languages or varieties of language that are freed from linguistic ambiguities.

Apart from this, T-comments can even pertain to radically different types of content as evidenced by

(S10) I have worked forty-eight hours.

Interpreted in a literal sense (S10) would convey the thought that the speaker has worked for forty-eight hours, presumably more or less without interruption. But by using a sentence like (S10) one can equally convey a 'conversational implicature' that could be paraphrased by

(S10*) I have completed an amount of work which would be normally achieved in (almost) double the time.

A T-comment expressing assent or dissent concerning (an utterance of) (S10) might not, and presumably would not, linguistically reveal whether it pertains to its literal import or to some conversationally induced import.

Considered in this light the following thesis should appear acceptable: if the existence of different types of content is admitted at all a T-comment can, in principle at least, pertain either to a linguistically indicated content or to a holistic content or to different specimens of linguistic or holistic content – without requiring any difference of its

(linguistic) meaning. A corollary of this tenet is that the number of theoretical possibilities can increase in correlation with the number of T-comments that function as (direct or indirect) objects of a T-comment.

Reflections on the argument II*) discussed in section II.B) support this contention. Assuming that (the truth ascription of) Ψ pertains to the linguistically indicated content of Ω and (the truth denial of) Ω to the linguistic content⁴⁷ of Ψ the situation is similar as the one that had been ascertained above regarding Σ . Both items indirectly dispute the truth of their own linguistic content. Given, however, that Ψ is taken as pertaining to the linguistic content of Ω , the latter item could be also interpreted as pertaining to the holistic content of its counterpart Ψ , claiming that Ψ , by attributing truth to the linguistic content of Ω and denying the truth of its own truth ascription, is bound to fail in any attempt to impart a truth. In this case it would be consistent to attribute the property of expressing a truth to Ω but not to Ψ . As a matter of fact, even Ω could be interpreted in a way that implies an inconsistency of its holistic content: Ω , by denying the truth of the linguistic content expressed by Ψ might entail a denial of its own truth denial. Instead of taking the increase of theoretical possibilities as a symptom of the untenability of the envisaged account, we consider it more appropriate to conclude that T-comments intrinsically exhibit a sort of indeterminacy which could be diminished by resorting to explanatory paraphrases, in particular expressions of the form ‘By affirming/negating that p, S expresses* something which is (not) true’.

Considering against this background the example dealt with in I), one can be tempted to object that even with respect to Σ one must concede the possibility that the item pertains to its own holistic content and thereby conveys a truth relying on the fact that it is inconsistent to deny the truth of its own content and simultaneously deny (the truth of) this denial. Even in light of the circumstance that it is rather unusual that one and the same item is used for making concurrently statements of comment pertaining to different levels, there is no need

⁴⁷ Henceforth the term ‘linguistic content’ abbreviates ‘linguistically indicated content’.

to categorically reject this possibility. If one admits this possibility one is bound to accept that one and the same T-comment possesses a plurality of holistic contents differing in truth value. This is, however, perfectly compatible with the stance that if a difference of truth value can be ascertained a difference of propositional content must be acknowledged too. The tenet, on the other hand, that one and the same linguistic item can exhibit a plurality of contents without presupposing linguistic ambiguity finds independent support with respect to T-comments in particular.

(7)

Against this backdrop, one can equally ascertain how it is feasible to make consistent truth denials with respect to the items appearing in a(n infinite) ‘Yablo-sequence’. The decisive point is that if such denials are couched in formulations, such as

All members of a Yablo-sequence are untrue.

All elements within a Yablo-series fail to express a truth.

the verbalizations must permit particular (sorts of) explications which cannot be equivalent to

That which all members of a Yablo-series express is untrue.

but only to

All members of a Yablo-series by expressing what they express express* something which is not true.

or more specifically:

All members of a Yablo-series, inasmuch as they express that that which all subsequent members express fails to be true, fail to express* something true.

The last formulation makes fully explicit that, given that all the members in the series negate the truth of the explicitly expressed content of all the subsequent members, they cannot exhibit a true holistic content. The reason is evidently that truth of any member

would require on the one hand that all subsequent members fail to express anything true and on the other hand that that which all the members express, namely that all subsequent members fail to express anything true, is untrue. This is inconsistent. Accordingly one can assign truth to a T-comment that gainsays the truth of the holistic content of the elements in a Yablo-series without denying the truth of their explicitly expressed content. Even the possibility exists that some 'meta-comment' exhibiting the described characteristics occurs within a Yablo-sequence. But this supposition would entail that its holistic content differs from that of those members implying truth denials with respect to the linguistic contents of the members subsequent to them. But a T-comment denying the truth of a true T-comment of this sort would entail an additional divergence regarding holistic content: It would amount to a negation of the truth of a T-comment which truly disclaims that an inconsistent holistic content cannot be true and has to be assessed as failing to be true on that account. Again the principle that deviance in truth value presupposes deviance of content is not violated. But the possibility of the existence of phrasings potentially embodying an incalculable amount of divergent contents which in principle can be accounted for by various explicatory paraphrases is an inevitable consequence of the proposed account.

Considering this, one can be prompted to object that at least with respect to T-comments arranged in an infinite Yablo-series the concept of a holistic content amounts to an absurdity. For illustrating the problem it suffices to consider the supposition that all denials of the individual elements exclusively pertain to the linguistic content of the subsequent members. Isn't it obvious that on that condition any member's being true by requiring that all subsequent members fail to be true presupposes that for every member some member exists which does (not not) express a truth? This means with respect to the first member M_1 in particular that, since the explicit content of all subsequent members must be untrue, some member M_m which directly or indirectly follows its subsequent member M_2 exists which expresses a truth; since this in its turn requires that the linguistic contents of all members subsequent to M_m are untrue some member M_{m+n} must occur

following the member which immediately follows M_m in the series that expresses a truth, and so on ad infinitum. Doesn't this amount to the supposition that M_1 possesses an infinitely complex holistic content and that the same holds true for any other member of the sequence? — Against this it can be retorted as follows: If the assumption of an infinite series of T-comments does not amount to absurdity, why should we suppose that the assumption of a content of infinite complexity amounts to an absurdity? If, on the other hand, the assumption of contents of infinite complexity is absurd, should not the assumption of an (actually) infinite series of T-comments also fall into discredit?⁴⁸ Anyhow, if a deviousness resides in the idea of a content of unlimited complexity there might exist an additional reason for refusing to attribute truth to the elements in a Yablo-sequence or for attributing absence of truth to them.

Leaving aside these questions one might still insist that the mere supposition of the existence of contents of extremely high complexity suffices for discrediting the hypothesis. If a type of content which allegedly plays a pivotal role for truth-value assessments can exhibit a degree of complexity which renders it unidentifiable for normal beings, it threatens to annihilate its supposed function. As a finite but extremely long Yablo-series could thwart all attempts to completely specify its holistic content, it might appear mysterious how any normal being should ever be in a position to definitely ascertain questions of truth or untruth concerning its elements. — It is not difficult to see that and why this criticism is unwarranted: It would suffice to consider only the first member of a series if one only knew that the subsequent members possess exactly the same import even ignoring how long the sequence is. Knowing that the first member disputes the truth of the explicitly expressed content of the subsequent members and knowing that the following member equally disputes the truth of the linguistic content of the members following it, one can definitely establish that the initial member must fail to be a truth, that the same must hold true

⁴⁸ If that were the case the situation envisaged in the Yablo-paradox would not satisfy the postulates stipulated for the recognition of genuine truth paradoxes.

for the second member if another member possessing the same import follows it and so on. If in any conjunction of propositions one detects only two elements which are mutually incompatible one can ascertain in advance that the complex conjunction must fail to be true irrespective of the identity of the remaining members. Hence full and comprehensive specifications of (holistic) content are not necessarily required for achieving categorical determinations of truth value.

Besides, even impossibility of categorical truth-value determination does not entail absolute ignorance concerning truth condition. This can be easily seen by considering a slight modification of the famous Epimenides paradox: If Epimenides the Cretan had said:

(S11) Nothing which any other Cretan apart from me has said is true.

one is surely not in a position to ascertain whether (S11) expresses a truth or not, as long as one does not know anything about utterances made by other Cretans. But one can definitely establish that *if* any other Cretan had said something true *then* (S11) cannot express any truth. For vindicating conditional truth assessments of that kind, understanding of the mere linguistic meaning can be sufficient even in the absence of knowledge of a holistic content. (This phenomenon deserves to be indicated in passing because it possesses significance for the methodology of empirical investigations of intellectual traditions.) Alternatively, one might say that linguistic understanding is apt to provide “truth-condition templates”, schemata like

If there is a *p* such that some Cretan other than Epimenides says that *p* and it is true that *p*, then (S11) expresses an untruth.

If there is no *p* such that any Cretan other than Epimenides says that *p* and it is true that *p*, then (S11) expresses a truth.

Schemata of this kind furnish guiding lines for evaluating possible contents with respect to their potential of verifying, falsifying, confirming or casting doubt on (statements made by) sentences like (S11).

(8)

The distinctive feature of the ‘genuine’ Epimenides paradox is that its starting point does not refer to specific sentences uttered or statements made by other people. In order to clarify the matter let us consider the situation against the background of the sentence

(S12) No proposition is true,
 in other words:
 Every proposition is untrue, or
 Every proposition is such that it is not true.

Let us take the term ‘proposition’ in the sense of either ‘proposition that is ever expressed’ or ‘proposition that has been expressed till the present time and including the present time’. The reason is that an assumption to the effect that no proposition is true can amount to an absurdity because it could be argued that for any untrue proposition there must be a negated counterpart which is not untrue or true. In the introductory section I of this essay we stipulated that assumptions of absurdities are not to be permitted as bases for the derivation of truth paradoxes. In contrast the suppositions that no true proposition expressed at some time exists or that among the propositions that have been formulated in the past and at present none exists which is true are probably, and hopefully, empirically wrong but not absurd or inconsistent. On the other hand, no restriction must be imposed on the range of the quantification. This means that (S12) is not equivalent to

No proposition which is not expressed by (S12) is true.

but can be elucidated by

Absolutely no proposition is true.

For facilitating an assessment against the background of ordinary intuitions the following chain of reasoning will be presented without employing uncommon technical symbols and the leftmost numbers signal (as often before) the assumptions on which pertinent lines depend, if there are any such assumptions:

X)

- | | |
|---|----------------------|
| 1 (1) No proposition is true. | A |
| 1 (2) The proposition that no proposition is true is not true. | 1 UE ⁴⁹ |
| 1 (3) It is not the case that no proposition is true. | 3 Tp/p ⁵⁰ |
| 1 (4) No proposition is true &
It is not the case that no proposition is true. | 1,3 &I |
| (5) It is not true that no proposition is true. | 1,4 RAA |
| (6) Ergo: There is some proposition which is true. | |

This outcome is apt to trigger the following consideration: As the proposition that no proposition is true cannot be true because it involves an internal inconsistency the result formulated in line (6) can only be interpreted as meaning that some true proposition exists which differs from the proposition that no proposition is true. As indicated above, it is highly probable that, as a matter of fact, some other true proposition has been articulated in the past. Nevertheless, the idea that the proposition that some true proposition has been expressed at the present time or before constitutes an *a priori* truth — after all, line (6) does not depend on any assumption — appears eccentric. Should we assume that it must be *a priori* certain that (S12), or some synonymous item, is not the first sentence that has been articulated in the history of the universe? In addition to this one can demonstrate that a parallel argument could be construed employing a sentence of the form

No proposition expressed in context C is true.

where ‘C’ represents a description which is so specific that at most one utterance occurs in the pertinent context. Accordingly, one should ask: What goes wrong in the argument presented in X)?

In the light of the principle that every proposition entails itself one can easily ascertain that the assumption of the first line in X) entails

⁴⁹ According to rule of universal quantifier elimination (UE). Note that the first line is to be understood in such a way that the proposition that no proposition is true has been expressed at the latest when the line has been written down.

⁵⁰ In accordance with the principle that ‘(The proposition) that p is true’ is equivalent to ‘p’.

both that no proposition is true and that it is not true that no proposition is true. Thus the truth of this contradiction can be regarded as a necessary requirement for the possibility of the truth of the proposition expressed in (S12). Using the terminology explained earlier it can be contended that the truth of the proposition that no proposition is true means* not only that the proposition that no proposition is true is true but also that the proposition that no proposition is true is not true. This means that the proposition's being true means* both that no proposition is true and that it is not true that no proposition is true. Given that assuming a proposition means* to assume that the proposition is true, one can claim that making the assumption formulated in the first line of X) means* to make the assumption that it is both the case that no proposition is true and the case that it is not true that that no proposition is true, in other words that it is both the case and not the case that no proposition is true. Hence taking into account and making explicit what treating the first line as an assumption means*, one obtains the following representation of the argument:

X*)

1 (1) No proposition is true &

It is not the case that no proposition is true. A

1 (2) It is not the case that no proposition is true. &E

1 (3) There is some proposition which is true.

The result obtained in (3) of X*) is not disconcerting because this line depends on the assumption of the first line. It only reflects the tenet that from contradictions propositions can be validly derived irrespective of whether the consequences are true or not.

It is not difficult to discern that a marked difference exists between that what it means* that a proposition is true, what a proposition's being true means* or, maybe, what a proposition means* on the one hand and on the other hand the question what a linguistic item expressing a proposition means. Evidently the expressions

‘No proposition is true.’

and

‘No proposition is true and the proposition (itself) that no proposition is true is not true.’

do not mean the same. Nevertheless, the latter expression can be regarded as articulating what the fact that the range of the quantification in (S12) includes the proposition expressed by (S12) means*. Hence the holistic content of (S12), which amounts to the content of the last sentence quoted above or of the first line in X*), yields an explication of the most important consequence of the pertinent character of the quantification or of its meaning*.

Those results facilitate an assessment of the ‘genuine’ Epimenides paradox that was presented as argument VI) in section II.D). It possesses the distinguishing feature that references to (TE2) or rules traceable to (TE2) are absent. As the individual derivational steps in VI) appear irreproachable it can be presumed that the problems lie elsewhere.

It is apparently taken for granted that the first assumption:

$$K[\forall x (Kx \rightarrow Fx)]$$

to be read as:

(The proposition) that everything which is asserted by a Cretan is false is asserted by a Cretan.

articulates a possible situation. It has to be taken in this way because otherwise the derivability of an incredible consequence would lose its paradoxical character. Here ‘K’ ought to be interpreted in the sense of ‘is asserted by a Cretan’ and not in the sense of ‘is said by a Cretan’, for it is not unproblematic to treat ‘assert’ and ‘say’ as interchangeable in this context.⁵¹ The verb ‘say’ can be employed in a way according to which ‘A Cretan says that everything asserted by Cretans is false’

⁵¹ In Künne 2013 the formula ‘ $\forall x (Kx \rightarrow Fx)$ ’ is paraphrased on p. 54 by ‘Everything that is asserted by a Cretan is false’ [Alles, was von einem Kreter behauptet wird, ist falsch] and according to p. 55 the first line of VI) should express that a Cretan says that Cretans only assert falsehoods [ein Kreter sagt, dass Kreter nur Falsches behaupten]. He clearly wants ‘sagt’ to be taken in the sense of ‘behauptet’.

would mean that a Cretan uses linguistic expressions which are synonymous with ‘everything asserted by Cretans is false’. Understood in *this* sense, the assumption that a Cretan says that everything asserted by Cretans is false certainly represents a possibility. But does this suffice for guaranteeing the possibility that a Cretan *asserts* that everything asserted by Cretans is false? Can it be taken for granted that by *assertively* uttering certain words nothing else is asserted than that which those words express?

According to line 4 in VI) the assumption that everything asserted by a Cretan is false entails that if a Cretan asserts that everything asserted by a Cretan is false then this, viz that everything asserted by a Cretan is false, is false. Hence the holistic content of sentences like

(S13) Nothing a Cretan asserts is true.

(S14) Everything a Cretan asserts is false.

is not the (linguistically) expressed content of those sentences but is equivalent to a content which entails

Everything a Cretan asserts is false, and if a Cretan asserts that everything a Cretan asserts is false then it is false that everything a Cretan asserts is false.

or in the notation employed in VI):

$$\forall x (Kx \rightarrow Fx) \ \& \ (K[\forall x (Kx \rightarrow Fx)] \rightarrow F[\forall x (Kx \rightarrow Fx)])^{52}$$

This formulation makes explicit the fact that the quantification in items like (S13) or (S14) is unrestricted; it articulates what it means* that the proposition intended to be expressed by those sentences is true. Thus the content of the assertions made in utterances of (S13) or (S14) does not coincide with their linguistically expressed content. If

⁵² Note that the submitted formulation presents merely an explication of the import of the universal quantification and on this account need not represent the holistic content of the pertinent sentence if it were uttered by a Cretan. As a matter of fact, whereas the formulation could specify the holistic content of sentences uttered by Non-Cretans, the holistic content of the sentence uttered by a Cretan — and the content of the assertion made by a Cretan assertively uttering it — can be more specific. More on this will be said below.

somebody assertively utters one of those sentences he asserts a proposition which is equivalent to the conjunction specified above. This means that in the pertinent situation it is not the case that nothing else is asserted than that which the words express. To be sure, under these circumstances the holistic content of an assertion can differ drastically from anything which someone employing a sentence ‘has in mind’. But against those who want to employ this fact for discrediting the account it should be maintained that exclusive orientation by what subjects have in mind when they linguistically express something generally amounts to poor methodology.

It is possible that someone utters the sentence:

(S15) It is extremely cold here now.

without possessing the slightest idea concerning his location or the precise date of his utterance. But this does not refute the tenet that it can be a most important task of interpretation to identify location and date against the background of an external or even objective perspective. At any rate, such identifications are vital for making decisions about the truth or lack of truth of what has been said, or rather, asserted.⁵³ For assessing the truth value of a T-comment formulated by

(S16) That is true.

it is indispensable to identify the proposition referred to. Among T-comments there is a special variety which could be christened ‘blank T-comments’. Somebody might assertively utter the sentence

(S17) Whatever the president answers to the question will be untrue.

and at the same time not know what the president in fact said or will say. The statement is made on the basis of the assumption that one can presume in advance that the president gives an untrue answer. It is at least not counter-intuitive to hypothesize that someone making a blank

⁵³ This need not be always the case if indexical elements are involved: With respect to the sentence

If it is Monday today it will be Tuesday tomorrow.

truth-value assessments can dispense with identifications of reference.

T-comment asserts something that transcends the linguistically expressed content. Accordingly, the proposition that it is not true that the president in question has never met the Russian ambassador would be part of the asserted content if the president were to say:

(S18) I have never met the Russian ambassador.

It is conspicuous that under such circumstances explorations concerning the query of what a speaker or author ‘really thought’ are irrelevant. For there is nothing in the mind of subjects making blank T-comments which could decide questions of truth. Instead of appealing to linguistic intuitions regarding the word ‘assertion’ one might rather allow that theoretical considerations play a critical role in the present connection: If one prefers, on theoretical grounds, to connect the concept of assertion and of asserted content most directly with truth and absence of truth then contents of assertions must be items which are not determined by linguistic meaning and mental facts.

Replacing

$$\forall x (Kx \rightarrow Fx)$$

in VI) consistently by

$$\forall x (Kx \rightarrow Fx) \ \& \ (K[\forall x (Kx \rightarrow Fx)] \rightarrow F[\forall x (Kx \rightarrow Fx)])$$

yields again the result that the content of the Cretan’s remark leads to an inconsistency and cannot be true — given that inconsistencies can never be true. On the other hand it can be ascertained that instead of the disconcerting consequence that (at least) some truth must have been asserted by some Cretan one now merely needs to admit a (fairly plausible) principle to the effect that if something is a truth then it cannot (simultaneously) be a falsehood if asserted by a Cretan.⁵⁴ The counterpart of the final line 11 in VI) which previously read:

$$K[\forall x (Kx \rightarrow Fx)] \rightarrow F[\forall x (Kx \rightarrow Fx)]$$

⁵⁴ Note that this does not imply a denial of the possibility that the very act of an assertion can make the asserted content false. Consider an assertive utterance of ‘At this very moment nobody asserts anything’.

If it is asserted by a Cretan that everything asserted by a Cretan is false, then it is false that everything asserted by a Cretan is false.

must now read:

$$\begin{aligned} &K[\forall x (Kx \rightarrow Fx) \ \& \ (K[\forall x (Kx \rightarrow Fx)] \rightarrow F[\forall x (Kx \rightarrow Fx)])] \\ &\quad \rightarrow \\ &F[\forall x (Kx \rightarrow Fx) \ \& \ (K[\forall x (Kx \rightarrow Fx)] \rightarrow F[\forall x (Kx \rightarrow Fx)])] \end{aligned}$$

If we dispense with abbreviations and the notation of formal logic this turns into a compound sentence which is not a jewel of English prose:

If it is asserted by a Cretan that

(both) (i) everything asserted by a Cretan is false and (ii) if it asserted by a Cretan that everything asserted by a Cretan is false then that (namely that everything asserted by a Cretan is false) is false

then it is false that

(both) (i) everything asserted by a Cretan is false and (ii) if it asserted by a Cretan that everything asserted by a Cretan is false then that (namely that everything asserted by a Cretan is false) is false.

Notwithstanding the deterring complexity, which merely results from the circumstance that all details are explicitly spelt out, one can clearly assess what the result amounts to. The formal structure reveals that, if a Cretan would assert that which he supposedly asserts, the content of his assertion would be false. But the proposition which is declared to false markedly differs from the previous conclusion. Declared to be false is a proposition to the effect that two things concurrently hold good, namely (i) that everything asserted by a Cretan is false, and (ii) that if a Cretan asserts that everything asserted by a Cretan is false it a falsehood that everything asserted by a Cretan is false. It is easy to recognize that the consequence does not any more possess the paradoxical corollary discerned before. As it is the conjunction of (i) and (ii) and not (i) alone which needs to be false or untrue, it cannot be deduced that something asserted by a Cretan is true. What really follows is something which intuitively appears completely plausible, namely the proposition that if everything asserted by a Cretan is in fact

false then this (viz. that everything asserted by a Cretan is false) is a truth (even) if a Cretan asserted (among other things) that everything asserted by a Cretan is false.

This outcome is completely compatible with the assumption that somebody can truly assert that everything asserted by a Cretan is false and that a falsehood is asserted if a Cretan asserts that everything asserted by a Cretan is false. After all, line 11 in VI) merely affirms that this cannot be truly asserted if a Cretan makes an assertion to that effect. This discrepancy can be ascribed to a disparity of holistic content generated in the two different sorts of situations. If a sentence of the form:

If (person) A ϕ s that p, then q.

— where ' ϕ ' is replaceable by verbs like 'say', 'assert', 'deny' etc. — expresses a truth and is (assertively) uttered by someone, then its holistic content usually amounts to the true conditional proposition (linguistically) expressed by that sentence. As long as, and only as long as the utterer is not A himself, his being related in a way designated by ' ϕ s' to the proposition designated by 'that p' does not entail (the truth) of the proposition designated by 'q'. Even if 'q' were replaced by 'not p' a true assertion can be made by the conditional sentence. By assertively uttering a sentence synonymous with

For any p: if A asserts that p then q / not p.

everybody who is not A can assert precisely that which the conditional sentence expresses. If, however, the same sentence were assertively uttered by A, the universal quantification creates the effect that the truth of the consequent proposition constitutes a requirement for the truth of the assertion. Given that the content of the assertion is its holistic content, the asserted content exhibits the form

$((A \phi \text{ that } p) \rightarrow q) \ \& \ q$

whenever A uses a sentence of the form of the first conjunct in order to make an assertion. Hence holistic contents and contents determined by (linguistic) meanings of items containing indexical elements

together with circumstances of employment share the property of dependence on context.

These results make it possible to build a bridge between truth paradoxes and *set theoretic paradoxes*. Again it is appropriate to begin with a question of meaning by asking what it means for a set to be a set of all sets which do not contain themselves as elements. A natural answer would be, that some U is a set of sets which do not contain themselves as elements if it can be specified by the following description:

For all sets x : if x is not an element of x then x is (a set which is) an element of U and if x is an element of U then x is (a set which is) not an element of x .

This formulation fails to reveal which verdict ought to be given about U itself, which is supposed to be a set, but, and this is more important, it does not make explicit, whether the universal quantification should be taken as restricted to sets different from U or whether it does not involve any limitation at all. It is not difficult to find a solution for overcoming this drawback. The following formulation would satisfy the desideratum:

For all sets x : if x is not an element of x then x is an element of U and if x is an element of U then x is not an element of x , and this holds equally good if x is identical with U .

or more explicitly:

For all sets x : if x is not an element of x then x is an element of U and if x is an element of U then x is not an element of x , and if x is identical with U , then if x is not an element of x then x is an element of U and if x is an element of U then x is not an element of x .

As propositions of the form: 'For all x : Fx and Gx ' entail 'For all x : Gx ', it can be derived that

For all sets x : if x is identical with U , then if x is not an element of x then x is an element of U and if x is an element of U then x is not an element of x .

By replacing the variable 'x', bound by the universal quantifier, by 'U' in the open sentence, one obtains:

if U is identical with U, then if U is not an element of U then U is an element of U and if U is an element of U then U is not an element of U.

This means nothing but:

if U is identical with U, then U is an element of U if and only if U is not an element of U.

This makes plain that the proposition can be true at best if U is not identical with U, and, given that everything which exists needs to be identical with itself, it follows that the requirement could be met at best if the set U does not exist — if one does not want to adopt the audacious tenet that sets in general are non-existent.

In view of the fact that it is commonly supposed that a set of all sets not containing themselves as elements cannot exist, one might wonder what is achieved by replacing the first less explicit wording by the more explicit variants. In reply to this one can say that the explicit variants enable one to localize the contradiction. It can happen that one first specifies some idea or concept, say the concept of a prime number which is greater than 31 and smaller than 37, and after having performed some investigation concerning the 'world of natural numbers' one detects that no item falls under the specified concept. By resorting to the explicit varieties of the specification one makes clear that the situation regarding the concept of a set of all sets not containing themselves is different. The contradiction is, so to speak, inbuilt in the concept itself. It emerges on the basis of a mere reflection about the question as to what the non-restricted nature of the quantification amounts to: It amounts to the postulate that exactly the same principle which regulates decisions of class membership concerning U for sets in general should regulate such decisions also with respect to U. One might say, that the considered paradox is

essentially not a problem of sets but a problem of meaning.⁵⁵ This in its turn can furnish an explanation — or partial explanation — for the intuition that an affinity exists between truth paradoxes and paradoxes of set theory.⁵⁶

(9)

It is a characteristic feature of the examples considered in section II.E) above that equally consistent alternative truth-value assignments can be made and the choice between them seems completely arbitrary. For an assessment of the situation it suffices to focus on the situation where two persons A and B mutually ascribe lack of truth to linguistic items produced by them.

If the property of not expressing something true is attributed to a linguistic unit that possesses a content, it is not reasonable to interpret it as disputing any content to that item. It must be presumed, at least by default, that the denial implies an ascription of untruth to some content. Regarding the example of two ‘colleagues’ mutually ascribing untruth to a linguistic item produced by the other, the most

⁵⁵ The problem would not arise in the same manner if it were supposed that the general way of specifying classes or sets should not be given by expressions of the form: ‘The (set of) x , such that Fx ’, but rather by ‘The (set of) x , such that Fx & for all y : if $y = x$, then x does not contain y as member’ or: ‘The (set of) x , such that Fx & for all y : if y is member of x , then x is not member of y ’. That would be legitimate, if the relation of set membership should be irreflexive or asymmetric; in this case the expanded formulations would merely make those facts explicit set membership should be irreflexive or asymmetric; in this case the expanded formulations would merely make those facts explicit.

⁵⁶ The explicit description amounting to:

U is a set of all sets which do not contain themselves as elements and which contains itself as element if and only if it does not contain itself as element makes merely manifest that the less explicit specification:

U is a set of all sets which do not contain themselves as elements (latently) exhibits a similar inconsistency as, for example:

U is a set containing only sets and containing words as elements.

natural assumption is that each remark is about the explicitly expressed content of the sentence token produced by the other speaker. Hence A, by expressing that no item expressing a truth is produced by B, can be taken as asserting that the content of B's remark to the effect that no item expressing a truth is produced by A, is an untruth. This in its turn implies that A's remark to the effect that no item expressing a truth is produced by B is a true remark, that the proposition he himself expresses is true. The same can be said *mutatis mutandis* about B and the linguistic item produced by him. Each of A and B, by ascribing lack of truth to the content of the remark of his colleague, claims truth regarding his own remark. It emerges as a result that the holistic contents are not exhausted by the implied denials. Rather, the sentence uttered by A possesses a holistic content to the effect that no item expressing a truth is produced by B *and* that he himself expresses a truth by saying that no item expressing a truth is produced by B. The sentence uttered by B possesses as its holistic content the conjunctive proposition that no item expressing a truth is produced by A *and* that he himself expresses a truth by saying that no item expressing a truth is produced by A. Assertive utterances always involve a claim of truth with respect to something expressed by them, but, or course, normally the proposition that one is expressing a truth is not an ingredient of the content of one's utterances. By assertively uttering the sentence

Napoleon Bonaparte made war on Prussia.

one asserts that Napoleon Bonaparte made war on Prussia but not that one asserts a truth by assertively uttering the sentence 'Napoleon Bonaparte made war on Prussia'. The content, be it linguistic or holistic, does not contain metalinguistic ingredients under these circumstances. But the situation differs in the case of the T-comments under consideration. Here one is entitled to contend that all pertinent items, i.e. both the sentence uttered by A and the sentence uttered by B, fail to express a truth. The crucial reason lies in the second component of the holistic contents according to which each of the two adversaries ascribes truth to his own remark. The reason for refraining from assigning truth to those ingredients is that no independent

objective support exists for such an assignment: Any conceivable ground for attributing truth to the linguistically expressed content which ascribes lack of truth to its counterpart relies on an attribution of truth to itself and generates a vicious circle. Hence the situation closely resembles situations in which a sentence attributes truth to itself or two sentences mutually ascribe truth to each other (and to nothing else)⁵⁷: There is no truth ground which does not entail the truth of the item to be evaluated.⁵⁸ (The idea of a set *S* which is such that it only contains itself as an element embodies a similar problem. If the set *S* contains only the set *S* as an element then the set which it contains as its element must contain the set *S* as its element and so on. How should it be possible to ascertain the identity of this set, given that the identity of sets depends on (the identity) of the elements they contain?)

An explanation of the possibility of correctly attributing absence of truth to both members of the pair was given before: A sentence expressing a denial of truth regarding the sentences produced by *A* and *B* as well as the statements made by them can be used in a way that it pertains to the holistic imports of those items. If a third person *C* expresses a denial of this kind he does not dispute the truth of the linguistically expressed content of the sentences uttered by *A* and *B*

⁵⁷ A linguistic unit can be true or express a truth only by being related to some content that is capable of being a truth. But a content to the effect that some linguistic item is true or expresses a truth is not by itself a content which can represent a truth. The fact that truths can be asserted by using expressions which merely express attributions or denials of truth regarding linguistic items does not yield a valid counterargument. Even by employing 'yes' or 'no' one can under appropriate circumstances assert something.

⁵⁸ The proposition

(i) that it is true that Napoleon made war on Prussia
does not entail the proposition

(ii) that the (English) sentence 'Napoleon made war on Prussia' is true.

For one thing, (i) could hold good even if no (English) sentence expressing this fact — or any fact — had ever been produced. More importantly, a derivation of (ii) from (i) would require (TE2) as an additional premise. (i) can be considered as an independent ground for the linguistic item's being true or expressing a truth.

but the truth of their holistic contents. C thereby disclaims truth to the disjunctive proposition that A expresses a truth and B an untruth or B expresses a truth and A an untruth. By denying such disparities C's denial is associated with a holistic content differing from the contents associated with the original remarks made by A and by B respectively. One might contend that no compelling reason exists for interpreting the sentences uttered by the two adversaries in the manner envisaged above. However, if one assumes that, for example, B's remark differs from the one made by A by the fact that it disputes the holistic content of its counterpart and that accordingly B expresses a truth whereas A does not, then the principle that differences of truth value entail differences of (some) content remains unaffected. If both remarks were directed against holistic contents in the same way, one avoids admission of any unfounded bias by dismissing truth regarding both. As a matter of fact, when confronted with truth denials one might not be in a position to definitely ascertain their holistic content. There might be nothing which could justify any categorical decision between different alternatives; so that one can at best attribute truth or untruth under various explicatory interpretations. If based on those considerations a denial of (categorical) truth is expressed, it possesses yet another holistic content.

V. Outlook

The account of truth paradoxes offered in this essay is conservative inasmuch as it does not question the validity of any elementary classical rule of derivation. The fact that our account does not involve a rejection of ‘Classical Logic’ does not imply any claim to the effect that Classical Logic gives a correct account of deductive reasoning in general. It does not imply either that non-conservative accounts of truth paradoxes are not worth pursuing. But the first segment of argument V) in section II above, which has the proposition that Napoleon Bonaparte was an orthodox Hindu in its final line, reveals that, as long as the general validity of two principles connecting meaning with truth is acknowledged, paradoxical outcomes emerge even if the classical rule of *reductio ad absurdum* is not appealed to. This makes it very natural to question the validity of these ‘non-logical’ principles.

On the other hand, the propagated stance of questioning (TE1) or (TE2) or both, together with [P1] and [P2], is quite moderate. So far, (TE1) and [P1] have not been contested. The censure was exclusively directed against (TE2) and [P2]. Even this criticism was modest because it discards (TE2) merely in the form in which it was originally envisaged. The distinction between linguistically expressed content and holistic content allows us to specify a relation between linguistic items and holistic content, which could be designated by the term ‘express*’. Then it turns out that both (TE1) and (TE2) could still be maintained with respect to the examples of truth paradox we considered if the term ‘express’ were consistently replaced by ‘express*’. (Admittedly, in actual practice it is often not required to signal the difference explicitly.)

At this point one might suspect that the revision is too restrained. Doesn’t it obscure problems that arise even with (TE1) and [P1]? Here

is an object of comparison. Allegedly, the French king Louis XIV uttered the sentence:

(19F) L'état, c'est moi.

Given that (19F) is synonymous with

(19E) I am the state.

one is entitled to assert that both the type sentence (19F) and the token of (19F) uttered by Louis XIV mean what (19E) means in English. But are we entitled to *derive* from this the following conclusion?

(*) If (19F) (as uttered by Louis XIV) expresses that I am the state and (19F) (as uttered by Louis XIV) thereby expresses a truth, then I am the state.

or:

(**) If Louis XIV said that I am the state and Louis XIV thereby said something true, then I am the state.

To be sure, a speaker assertively uttering one of these sentences can be credited with saying something true. But it is clearly inappropriate to affirm that the truth of (*) or (**) can be derived from the supposition that Louis XIV uttered a sentence which means (in French) the same as 'I am the state' (in English). By assertively uttering (19F) the French monarch did definitely not *intend* to say something which entails that any vintner in the Provence would express a truth if he were to utter a sentence which means exactly the same as (19F), nor did the king say anything a vintner might say by using a sentence possessing the same meaning as the sentence used by the monarch. What can be legitimately derived is only that Louis XIV by employing (19F) created a situation which can be correctly described by

(†) Louis XIV said / uttered a sentence which expresses that he himself, Louis XIV, is the state.

It should be obvious that the import of the expressions 'say' in [P1] and 'express' in (TE1) can only be equivalent to the import of those

terms in ($\dagger$), – an import which does not license the substitution of (19E) in the that-clauses, even though (19E) has the same linguistic meaning as (19f). Now this observation confirms our denial of the claim that the possibility that a Cretan employs words *meaning the same as* ‘no Cretan ever asserts anything true’ implies the possibility that any Cretan *asserts* that no Cretan ever asserts anything true.

Nevertheless, there remains a difference between (TE1) and (TE2). The latter is acceptable as a general principle at best if a relationship connecting linguistic items with holistic imports is hypothesized. The former is valid even in the version where ‘express’ is not substituted by ‘express*’. The same holds good, *mutatis mutandis*, for [P2] and [P1].

It might be contended that our account of the truth paradoxes fails because it postulates contents as bearers of absolute, not relativized, truth values that do not comply with the intuitive principle of linguistic compositionality which can (informally) be conveyed by the slogan

The meaning of a complex expression is determined by the meanings of its (immediate) constituents and the mode of their composition.

But even if the principle of compositionality as conveyed by the slogan holds for *linguistic* meanings (which is by no means obvious), our account of the truth paradoxes implies at most that there is *some* kind of content that does not comply with compositionality as conveyed by the slogan, and this is something that should be accepted on all sides. The so-called *conversational implicatures* of an utterance are not strictly determined by the conventional linguistic meaning of the sentence used in the utterance. This observation does not by itself militate against the thesis that the items conveying conversational implicatures also possess a ‘literal sense’ which *is* compositionally determined. According to our account, the same holds *mutatis mutandis* for *holistic meanings* and T-comments whose truth evaluation requires appeal to such meanings.

The account of the truth paradoxes recommended in this essay may seem to require rejection of the biconditional truth schema

s is true in L if and only if p

where the place-holder ‘s’ is to be substituted by a (revealing) designator of a declarative sentence in L (e.g. its quotation name) and the place-holder ‘p’ is to be replaced by a *translation* of that sentence into the metalanguage (English, in our case) that is employed in the biconditional.⁵⁹ Now the felicitousness of employing ‘is true’ as a predicate of sentences is questionable, and it was implicitly questioned above. But it is more important to note that it is far from obvious that our account entails a rejection of the biconditional truth schema. First of all, it requires at most a restriction to those truth candidates that are not self-referential T-comments. Second, it is not even clear that it demands such a stricture. If one applies the biconditional truth schema one invokes the notion of translation, and one can argue that the schema can be applied even to self-referential T-comments if one invokes an appropriate concept of translation. Using the terminology employed in this essay one can say that in the realm of T-comments adequateness of translation does not require identity of sense (linguistic meaning or linguistic meaning plus contextual parameters) but rather identity of holistic meaning. In this way the more comprehensive issue of the nature of translation comes into play, and one comes to recognize that the topic of the paradoxes induced by T-comments turns out to be part of a topic that is relevant for the humanities in general.

Not only in ordinary contexts but even in scientific contexts T-comments exhibiting some sort of self-reference play at best a marginal role. This might be taken to show that the distinction between one kind of meaning that pertains to linguistic items and another kind of meaning that pertains to entities which can be designated by expressions of the form ‘A’s being F’ and ‘that A is F’ possesses at best quite limited relevance. A preliminary move to dispel this

⁵⁹ As is well known, this schema is part and parcel of ‘Convention T’ (formulated by Tarski), according to which a formally correct definition of ‘true’ for a given object language L in some metalanguage (e.g. English) is materially adequate if and only if it implies all sentences which can be obtained from the schema by appropriate substitutions. — Cf., among others, Sainsbury 1988:139 or Kühne 2003:183.

suspicion consists in pointing out that at least the general distinction between meaning pertaining to linguistic items and meaning pertaining to non-linguistic items can certainly not be dismissed as negligible. After all, the realm of meaning pertaining to non-linguistic items is not exhausted by meaning pertaining to entities denoted by terms of the two forms mentioned above. It encompasses material objects, processes and events such as an accumulation of clouds that means rain, actions and kinds of behaviour like remaining silent in reaction to a question, and states of affairs like the fact that smoke is to be seen on the hill. Furthermore, substitution instances of 'A's being F' or 'that A is F', where 'F' is replaced by 'true' show that issues pertaining to non-linguistic items of this particular kind are far from being marginal: if one were to dismiss such issues wholesale as irrelevant one would declare questions of truth to be irrelevant.

According to our analyses of various truth paradoxes, posing questions as to what it means* that some item *s* expresses a truth is apt to reveal highly important and significant facts about them. They are important because they concern conditions under which a content related to linguistic objects can be assessed as true or untrue. They are significant because they are not always revealed by the linguistic meaning of expressions. The fact that for items, such as Λ , Σ or Ψ above, their expressing a truth means* — or would mean* — that certain contradictions are true cannot be discerned by merely inspecting their linguistic meanings. Reflecting on the explicitly communicated contents of Γ or Δ or grasping the linguistically imparted significance of the individual assumptions formulated in lines (1)–(6) in argument V) in section II.D above does not enable us to recognize that for Γ as well for Δ in those contexts being true means* that particular historical states of affairs obtain which may never have obtained at all. The importance and the significance of explications which answer questions as to what something's being true means* is not restricted to cases where such explications are indispensable for elucidating contents which can be evaluated in the dimension of truth and untruth without leading to inconsistency. In the realm of *historically oriented studies* — a realm that we shall enter in the next

essay — the relevance of posing such questions is to a large extent due to the widely distributed phenomena of equivocation and vagueness we encounter in documents of the past. If we wonder whether a formulation we find in such documents represents a (possible) truth or which condition could bestow truth on its content, an explication may be required which just as much transcends the realm of information discernible by ordinary methods of linguistic and philological analysis as do explications revealing truth conditions of linguistic items generating paradoxes of truth. This fact alone makes it imperative to take a firm stand against those who contend that explications which transcend both linguistically conveyed imports and actual intentions of language users are to be eliminated from assessments of traditional doctrines, teachings, theories, or tenets. After all, it is incontestable that in assessments of truth regarding controversial doctrines etc. upheld by our *contemporaries* high importance often accrues to questions as to what they mean*, irrespective of what the words and sentences by which they are expounded mean or what their proponents ‘have in mind’. When doctrines etc. advocated in the *past* are at stake the situation does certainly not differ to such an extent that it would be reasonable to neglect questions of truth and of meaning* entirely.

Admittedly, assertions apparently claiming their own falsity, predicates allegedly satisfied by predicates which do not satisfy themselves or terms seemingly referring to sets which contain as elements all sets that do not contain themselves as elements are fairly ‘exotic’. But the considerations they call for, considerations about what it means for such items to be true or satisfied or referential, are not only relevant to such outlandish examples. Awareness of the difference between the task of ascertaining linguistic matters of meaning and the task of ascertaining truth and meaning* is also of vital importance when teachings propagated in the course of history are at stake, and the latter task should not be neglected in this field either. Or so I shall argue in the second essay contained in this book.

Appendix

The so-called natural deduction techniques used in this essay stem in essentials from Gentzen, and the style of presentation follows Lemmon 1965. The lines of a derivation are numbered consecutively, '(1)', '(2)' etc. To the *left* of each line the numerals refer to the assumptions (if any) on which the formula in that line depends. Assumptions are premises not depending on any other premise in the derivation. To the *right* of each line acronyms designate the rule justifying the appearance of the formula at that stage and numerals designate the line(s) containing the premise(s) to which the rule is applied.

The following rules of derivation are pertinent in our inquiry:

1. (Rule of) Assumption (A). It allows the introduction of any proposition at any stage of a proof. We write to the left the number of the line itself in which the assumption is made, for it depends on nothing but itself.
2. (Rule of) '&'-Introduction (&I). It allows the derivation of the conjunction 'p & q' if both conjuncts 'p' and 'q' are given as premises. The conclusion depends on the assumption on which the premises 'p' and 'q' depend.
3. (Rule of) '&'-Elimination (&E). It licences the derivation of 'p' as well as of 'q' from their conjunction 'p & q'. Each of these conclusions rest on the same assumptions as the premise.
4. (Rule of) *modus ponendo ponens* (MPP). It allows the derivation of 'q' from the premises 'p $\rightarrow$ q' and 'p'. The conclusion depends on the assumptions on which 'p' and 'p $\rightarrow$ q' depend.
5. (Rule of) *modus tollendo tollens* (MTT). It licenses the derivation of ' $\neg$ p' from the premises 'p $\rightarrow$ q' and ' $\neg$ q'. The conclusion depends on the assumptions on which 'p $\rightarrow$ q' and ' $\neg$ q' depend.

6. (Rule of) Conditional Proof (CP). If 'p' is the assumption, or one of the assumptions, on which the derivation of 'q' depends, one is entitled to derive in the next step the conditional proposition ' $p \rightarrow q$ '. This conclusion no longer depends on 'p' but only on the other assumptions (if any) on which the derivation of 'q' was based.
7. (Rule of) *reductio ad absurdum* (RAA). If from the assumption 'p' or from several assumptions ' p_1 ', ' p_2 ', ..., ' p_n ' a contradiction, that is, a formula of the form ' $q \ \& \ \neg q$ ' can be derived, then we may derive the negation of 'p', that is, ' $\neg p$ ', or the negation of one of the several assumptions, e.g. ' $\neg p_2$ '. The conclusion no longer depends on the assumption it negates but only on the other assumptions (if any) on which the derivation of ' $q \ \& \ \neg q$ ' was based. Presupposing, reasonably enough, that a (genuine) contradiction cannot be true and that the rules appealed to in its derivation are deductively correct, the assumption, or one of the assumptions, on which its derivation depends has got to be false. If the derivation depends on several assumptions, RAA does not tell us which assumption should be negated.
8. (Rule of) Universal Quantifier-Elimination (UE). This is the only rule involving quantifiers that was appealed to in the preceding investigations. It licences substitutions of terms designating individual objects for variables bound by universal quantifiers. The singular conclusion rests on the same assumptions as the universal premise. If something is true of all objects (in a domain) it must equally be true of every individual object (in that domain).
9. (Rule of) Theorem Introduction (TI). This rule permits us to introduce, at any stage of a derivation, a theorem already proved or a substitution instance of such a theorem. In relying on a theorem an argument does not rely on an additional assumption, for a theorem rests on no assumption. — TI is not a basic rule of derivation: it merely enables us to shorten lengthy derivations that only appeal to rules like the eight preceding rules. (TI was implicitly invoked in lines (7) and (8) in argument V) in section II.D)

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